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Warped Ruled Surfaces of Historical Stone Bridges in Prague

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Abstract
This research concerns the use of warped ruled surfaces in the load bearing structures of 19th century stone bridges in Prague. The descriptions of a skew arch used at The Negrelli Viaduct and a special surface called "corne de vache" used at The Legion Bridge are based not only on historical survey, archival materials and period publications, but also on contemporary measurements.

Keywords Stone bridges · Ruled surface · Oval arch · Descriptive geometry · Modeling

Introduction

The importance of bridge construction has been very nicely captured by professor A.V.Velflík, Czech bridge engineer and professor at the Czech Technical University in Prague:

> "*Bridge building both today and in the past represents the utmost accomplishments of civil engineers, being a result of thorough studies in many areas of preparatory disciplines such as the theory of elasticity and strength, structural mechanics, graphic statics, stereotomy, and, of course, to the highest extent, mathematics; yes, today's amazing and generally admired accomplishments in the area of arched bridges are a result of a profound theory supported by many years of experience, ingenuity and boldness of civil engineers."* (Velflík 1921:91–96).

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The research follows up on the complexity of bridge building in the same manner and shows the importance of using warped ruled surfaces in load bearing structures of two stone bridges over the Vltava (Moldau) river in Prague. Particular attention is focused on stonecutting.

The first is the Negrelli Viaduct, a railway bridge, which was completed in 1850. The Negrelli Viaduct held the prime position for the longest viaduct in Europe until the year 1910. The length of it was 1110 m. The bridge had to span *Křižíkova* street at another angle than a ninety degrees angle. Due to this, warped ruled surfaces had to be used in the construction of the bridge.

The second stone bridge in this study is the present-day Legion Bridge finished in 1901. Here we could find a special warped ruled surface named "corne de vache".

Skew Arch in The Negrelli Viaduct

Warped Ruled Surface Used in The Negrelli Viaduct

The situation when faces of the arch were not being perpendicular to its abutments, the plan view was parallelogram, was solved with warped ruled surface. The directrix of the surface are two identical circles k*(S; r), $\bar{k}(\bar{S}; r)$* lying in parallel planes, ρ and $\bar{\rho}$. Line segment $S\bar{S}$ is not perpendicular to the planes of circles. The third directrix is line p, which passes through point Q, which is at the centre of the $S\bar{S}$ segment. Line p is perpendicular to the planes ρ and $\bar{\rho}$. (Fig. 1)

This skew arch classifies as a quartic surface (i.e. it is defined by an equation of degree 4) (Kadeřávek, Klíma, and Kounovský 1932: 823–827). The surface is centrally symmetrical according to the Q point, the plane of symmetry of the surface is the plane determined by the line p and the centres of the directrix circles.

Negrelli Viaduct

The Negrelli Viaduct (formerly known as the Karlín Viaduct) was completed in the year 1850 as part of the Project of the Northern State Railway of Olomouc—Prague—Dresden. At that time, it had a total of 87 stone arches, including 76 semicircular vaults, 8 segmented vaults, 1 skew arch and 2 pedestrian passages. The length of 1,110 m put it in first place in Europe. The designers were Czech railway engineer Ing. Jan Perner and Alois Negrelli, Tyrolean civil engineer, in the past the assistant of Lesseps in construction of the Suez channel. (Dudák and Rýpar 2020: 290–297)

The Negrelli Viaduct has been reconstructed during 1917–2021. As part of this reconstruction, the bridge was carefully documented. Before the reconstruction, a historical survey of the building was made. This survey, however, revealed a text

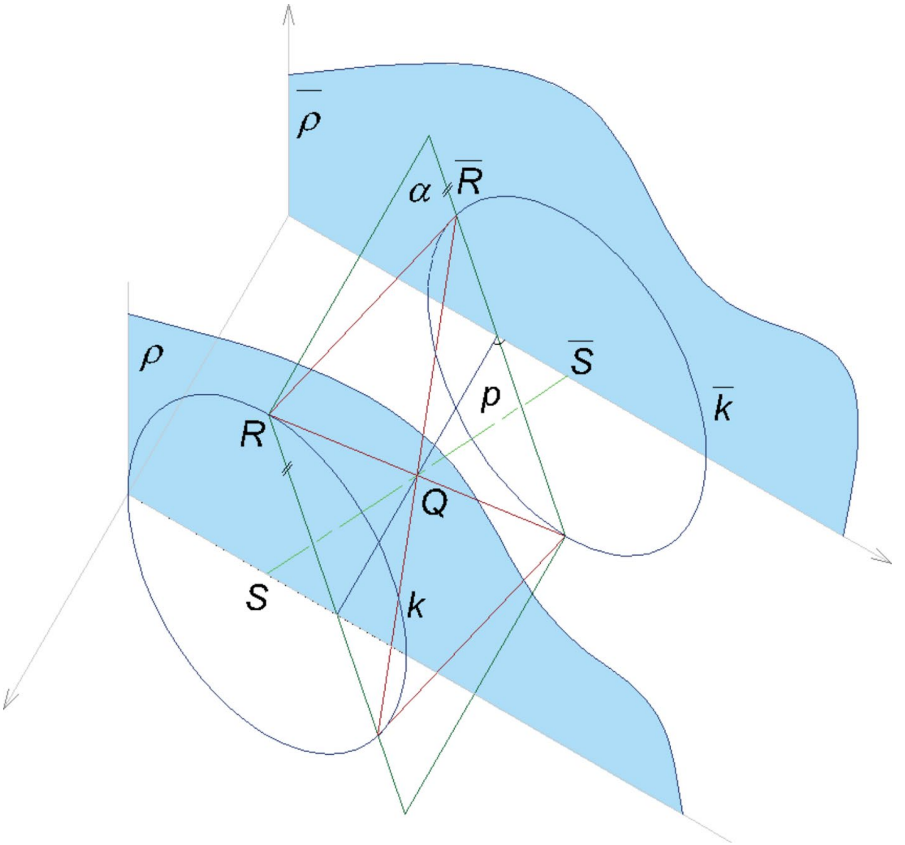

Fig. 1 Warped ruled surface—skew arch

description of the construction but no original plans of the said construction. These texts show that the skew arch has not been specifically addressed. It seems to have been more of a technical problem, which was subsequently solved by the building engineer. The advantages and disadvantages of the different methods which have been employed in the construction of skew arches were known at that time, such as the orthogonal bond and helicoidal bond. The builder of the viaduct chose an ordinary skew arch of the "biais passé" type as the best solution. The directrix—circles have a radius of 9.35 m and their theoretical centres lie below ground level.

Furthermore, the stonecutting aspect is also very important. During the reconstruction, some of the arches were replaced with new ones. The individual stones were scanned with a 3D scanner and then models were made from polystyrene, and subsequently carved from stone.

Figure no. 2 shows the skew arch of the Negrelli Viaduct.

Fig. 2 Negrelli Viaduct

The "corne de vache" Surface and the Present-day Bridge of Legions

The "corne de vache" Surface

The surface used in the load-bearing structure of the Legion Bridge in Prague is also a warped ruled surface. Since the bridge's design plans have been preserved, it is easier to study. The vaults are extended into the facade by a surface called the "corne de vache".

The directrixes of the surface are as follows: a circular segment to the k (S; r) front of the bridge arch, an elliptical arc formed by a cut in the arch. The ellipse is replaced here by a multicentre oval with consecutive circular arcs m1, m2, … The third directrix is a line p always passing through the centre of the circular arc, which replaces the ellipse at that point. Line p is perpendicular to the front of the arch (Fig. 3).

Replacing an ellipse with an oval is appropriate in terms of statics reasons. Determining the normal in this case is simple. This solution can already be found in the elliptical bridges of Jean-Rodolphe Perronet, French architect and structural engineer, known for his many stone arch bridges. (Gomez-Collado 2018)

The Legion Bridge

Today's Bridge of Legions was the fourth stone bridge built across the River Vltava in Prague. It was completed in 1901. The design by the architect Antonin Balšánek,

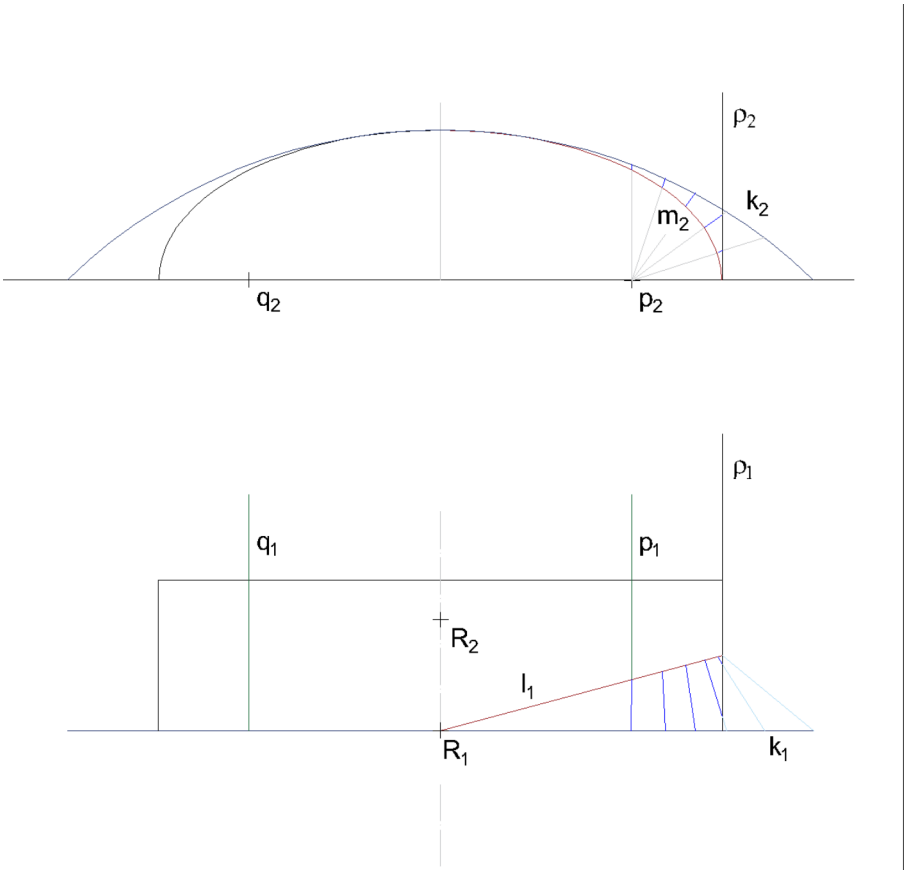

Fig. 3 Warped ruled surface—corne de vache

Czech architect and professor at the Czech Technical University, and the civil engineer Jiří Soukup, Czech bridge engineer, won the public competition. In the description of the design we find that the inspiration was the French bridge d'Alma over the River Seine, completed in Paris in 1856. The same type of arching, low elliptical arcs and the use of the "corne de vache" surfaces are evident at a first glance. (Štoncner 2019)

The research looks at the detailed design of ovals in individual vaults, followed by the design of the surface of "corne de vache" based on preserved period drawings.

In addition to archival materials, references are processed in the professional press of the time (Fig. 4, 5).

The function of the "corne de vache" surface is purely aesthetic in the case of this bridge.

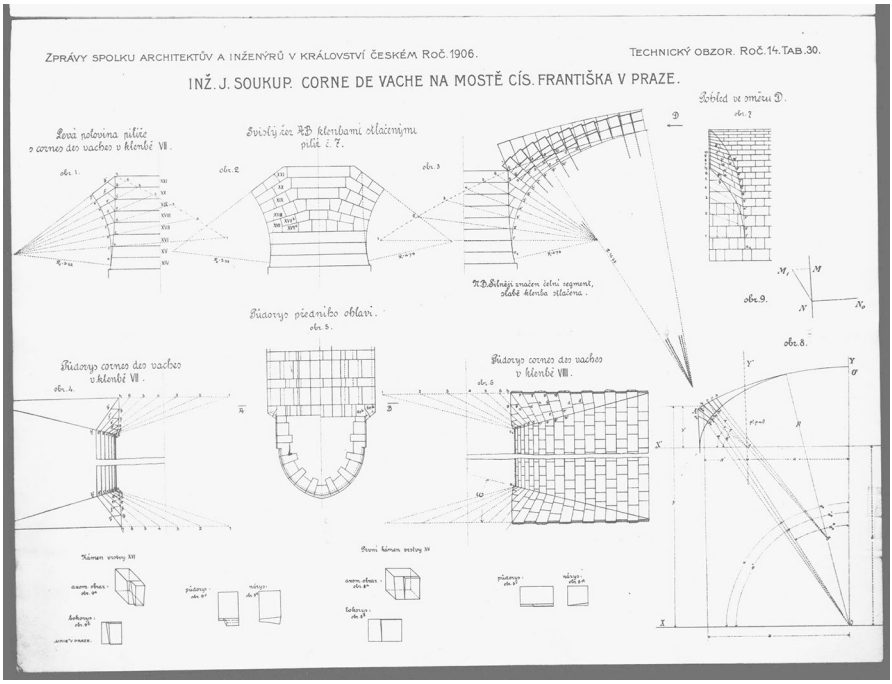

Fig. 4 Legion Bridge—construction of corne de vache

Conclusion

Research on these two stone bridges in Prague suggests that the way of using warped ruled surfaces in the load-bearing structures of the bridges is dependent not only on the design project but also on the technical implementation of the construction. Both ruled surfaces are typical for bridge arch structures, but their significance differs greatly. The skew arch is a necessary technical solution for a complicated situation. On the other hand, the "corne de vache" surface in this specific example is more of an aesthetic element.

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Conflict of Interest The corresponding author states that there is no conflict of interest.

Fig. 5 Legion Bridge, Prague

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Archives

Archive STK in Prague, V 020 Most Legií
National Technical Museum Archive in Prague, Fund 29 Bridges
The Archives of the National Technical Library

Dana Kolarova was born in Prague, Czech Republic. She studied Mathematics and Descriptive Geometry at the Faculty of Mathematics and Physics of the Charles University in Prague. Nowadays she is a PHD candidate and senior lecturer at the Faculty of Architecture of the Czech Technical University in Prague. The topics of her thesis is Geometry of load-bearing structures of historic bridges. She was awarded the gold medal of CTU for the extraordinary contribution in teaching descriptive geometry at the Faculty of Architecture of CTU and the personal creative efforts that lead to increasing the prestige and promotion of the topic of geometry in the teaching of architects in 2021.