Nexus Network Journal
https://doi.org/10.1007/s00004-023-00694-4

CONFERENCE PAPER

Origami and Descriptive Geometry: Tangible Models to Enhance Spatial Skills

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Accepted: 14 March 2023

Abstract
The use of tangible models to enhance spatial skills and geometric thinking is common practice in architects' and mathematicians' *curricula*. Thanks to its design/folding, origami is a powerful tool for transversal didactic experiences between drawing and geometry, where we find the ideal context in which to verify its effectiveness.

Keywords Descriptive Geometry · Modeling · Didactics · Algebra · Geometry

Introduction

Origami is a tangible expression of descriptive geometry. A sheet of paper is transformed into 2D/3D object (model) by a folding sequence. The process of manipulating an origami develops spatial visualization skills: it enhances the construction of relationships between visible shapes and the actions that create them, fueling the spatial prefiguration abilities. Straight folds are the result of intersection between planes and allow the overturning of a portion of plane. Conversely, curved folds require reasoning around their developments.

Therefore, the origami design makes use of the tools of descriptive geometry, creating links between geometric and visuo-spatial thinking. It leads to the construction of a geometric language shared between drawing and mathematics (Cumino, Zich, Pavignano 2022, pp. 99–100). So, it is also a useful tool for teaching mathematics at different levels, declining contents and languages in respect of contexts and users (Meyer, Meyer 1999).

The contribution illustrates the fruitful intersections between geometric thinking and paper folding in designing tangible models presented during the first edition of

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Published online: 28 March 2023

the International Spring School of Mathematics of Namibia NAISSMA 2022 (Windhoeck, Namibia).

The Research

Spatial visualization skills are the basis of the training of both the architect and mathematician *curricula*. The ability to 'see' shapes and translate them into representations (graphic, plastic and/or analytical) is both the structure and the outcome of geometric thinking. Within the modeling process of an origami each fold represents part of the structure which leads to the final model, it does not necessarily represents the outcome of the activity. Geometric thinking is part of every fold, therefore is the basis of the design and modeling process.

The culture of origami model sharing, coupled with the folding action, offers itself as an ideal context for building knowledge through 'routines' of the mathematics learning process as described by Lavie et al. (2019). Even Montessori in *Psychogeometry* (2018) used routine processes for interacting with physical models. She stated that the act of reworking an object as «keeping it in front of the senses, repositioning it in continuous, reproducing it with sensible images (drawings, paintings, paper works, etc.)» allowed the mind to obtain information from it. So, origami modeling not only offers a tangible geometry, but also the reiteration of the folding process, therefore an operational routine that becomes cognitive. Many other studies, ranging from Sundara (1893) to Friedman (2018) reveal the potentialities of origami in mathematics education. Nonetheless, our focus is on: the design of the model, how much geometry is part of the model conception process and how it is possible to make this practice part of mathematics training path to improve spatial visualization skills.

NAISSMA Context. The First Namibian International Spring School in Mathematics

The Mentoring African Research in Mathematics Programme (MARM), supported by the International Mathematical Union (IMU) and the London Mathematical Society in collaboration with the African Millennium Mathematics Science Initiative (AMMSI), promotes the strategic role of mathematics for the development of depressed areas. Led for the Politecnico di Torino by Prof. Letterio Gatto (mentor of the Department of Mathematics of the University of NAMIBIA), this was the ideal context within which to verify the effectiveness of the tangible approach of the geometry of origami models, to integrate practical problem-solving with visual strategies.

These activities involved a heterogeneous audience: participants of the NAISSMA and secondary school pupils.

Methodological Approach

On intuitive geometry «turning to the real world, following a constructive methodology, can also be achieved by focusing student's attention to real problems […], so that he immerses himself in a complex situation, close to those that occur in nature; and he […] will be led to analyze it, thus passing from the global to the element» (Castelnuovo 1962, p. 203). Still, 2D and 3D modeling exercises have been proposed both as specific situations and as an example of general cases. Origami therefore aims to «exercise the mind in two opposed processes: that of synthesis, which starts from the element to build, to arrive at a global, and that of analysis, which starts from a global complex situation and arrives at the element» (Castelnuovo 1962, p. 203). Thus, the origami model can assume multiple communicative values that determine the correspondence between shape and representation: when it has a symbolic value, the model is released from the concept of rigor, instead, when it assumes a descriptive value, its fold sequence becomes an operating procedure and the very essence of the shape itself.

Creating an origami model that represents a geometric shape while respecting its design rigor turns out to be a complex practice, to be mediated between a theoretical approach and material production, tools and techniques.

Defining methodological and procedural models dealing with real applications is a process that we could refer to as prescriptive mathematical modeling, a cognitive strategy starting from data (and relationships between them) to be projected in the perspective of established goals (Blum, Niss 2020).

In the contextualization of origami modeling, the quantification and systematization of the data needed for the solution of the real problem is directly related to its production, because the model is not only the theoretical solution of the problem. The origami model is conditioned by the material (thickness, composition, foldability) and by its dimension, therefore the solution described as a folding sequence must necessarily be completed with these data.

Tangible Models in Modeling Perspective: Descriptive Geometry as Connections Between Geometric and visuo-spatial Thinking

The development of a cube contained in a catalogue of models of mathematical surfaces supports the description of complex surfaces of four dimensions (Fig. 1a). It explains how this development does not allow modeling without further aids and

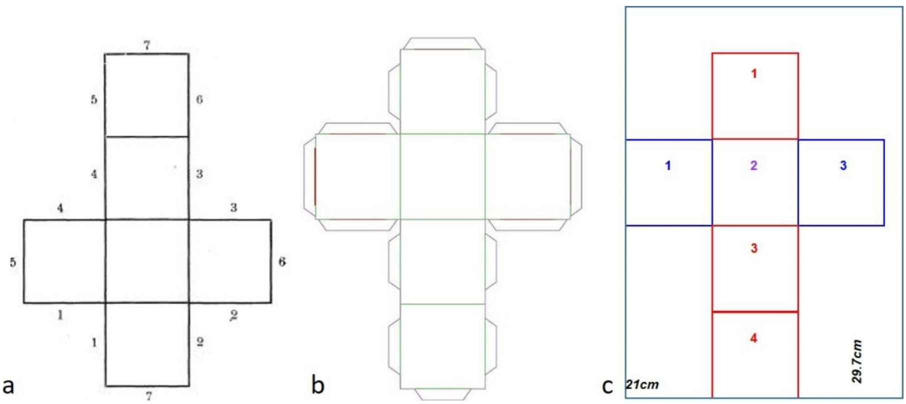


Fig. 1 Genesis of an origami model of a cube. (a) inspiration (Schilling 1911, p. 91); (b) developed cube produced with paper work (Cumino, Pavignano, Zich 2022, p. 255); (c) project of the designed origami cube fitting the A4 sheet

shows how the construction of a cube with paper passes from its development with the insertion of a closing system (Fig. 1b). This example suggests the first proposed exercise. It is about modeling a cube with an A4 sheet (21×29.7 cm), starting without any graphic/measuring reference.

The logic that underpin the construction of the model starts by analyzing the shape of the sheet and the 'needs' of the model. We need 6 square faces, linked together in order to construct alignments as in Fig. 1c. To close the model, we need at least an additional surface. We then proceed to represent these faces on the sheet surface by placing 3 aligned elements on the 21 cm side. In this way, we find the maximum dimension (constrain) of the cube's side to be 21:3 = 7 cm. We do not know how to divide a segment into 3 equal parts without proceeding with direct measurement. We must remember the need to add a further surface for the model closure. We place the closing surface in the alignment of the 4 elements and use this surface to solve the calculation of the length of the side of the cube: if we divide the 29.7 cm side by 4 we obtain a side of 7.425 cm, which is not compatible with the overall dimension of the cube. We need to make a first fold to create the closure system of at least 1.7 cm such as to bring the remaining length to be divisible by 4, obtaining a value of less than 7 cm (design constraint). Producing a crease of at least 1.7 cm means about 2 fingers wide (as we do not need rigorous dimensions). We then proceed with the modeling according to a folding sequence which leads to halving the measurements by folding the medians of the rectangles and managing the extra paper by creating bisectors of right angles (Fig. 2). This set of simple folds allows every kind of user to create the model. The result is consistent with the purpose of the activity, the construction of a cube with an undefined side. The result is not rigorous since the measurements of the sides are made by overturning the information through folds which are affected by the manual action; nonetheless, modeling from a drawn support would have the same limit as any other production. We proposed other exercises concerning origami representation of textual descriptions of relationships between flat figures contained in Montessori's psychogeometry. She made use of static wooden or cardboard models (Fig. 3). The next models are designed to be dynamic, leading users to see the relationships between the parts. This process, when repeated, can generate knowledge.

We created a dynamic model whose fold sequence can be re-proposed, triggering an infinite sequence to produce a fractal model (Zich 2019).

Figure 4 shows a monochrome model: here the movement allows to open and close the related elements. Figure 5 represents a polychrome artifact where the different colors of the two sides of the sheet come to highlight flat figures and relationships between the parts. Figure 6 shows a colored and dynamic model illustrating the Pythagorean Theorem. The folding sequences have been partially optimized to be shared by different users.

Conclusions

These experiences suggest some considerations on the relationship between theoretical surfaces and their tangible representation. Physical modeling introduces the possibility to apply some tools specific of architect's word into a mathematical context,

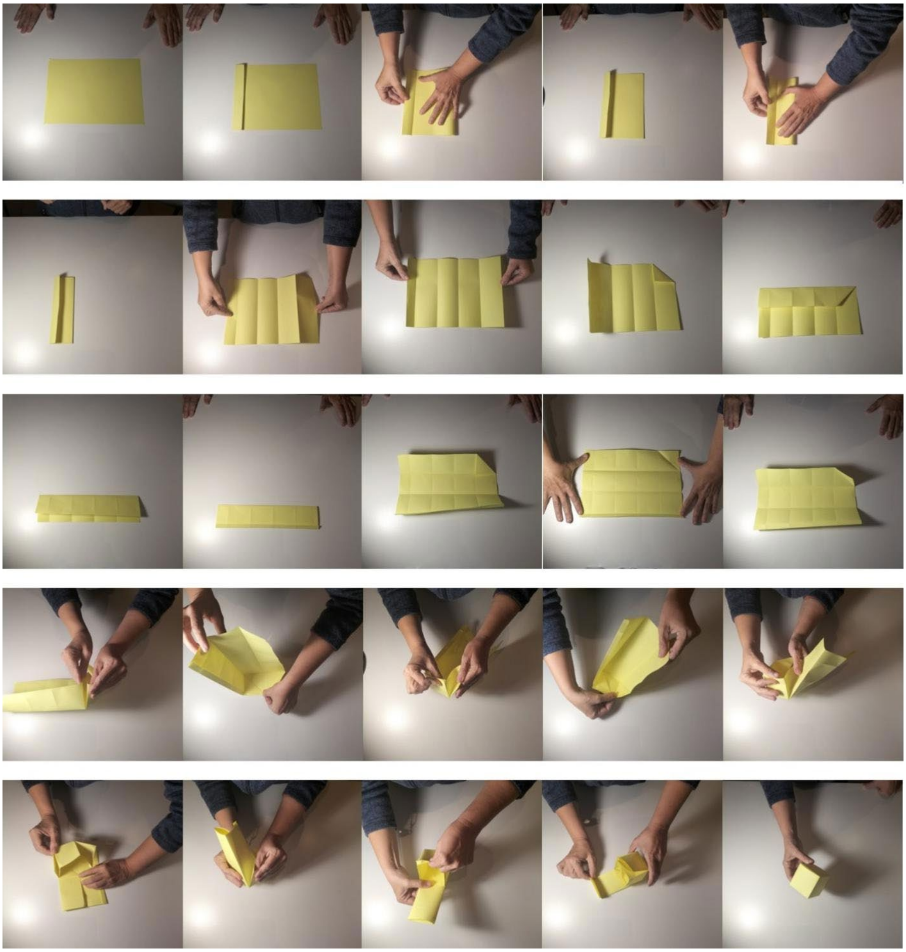
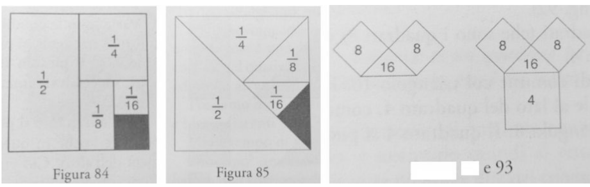


Fig. 2 Origami cube folding sequence

Fig. 3 Paper model for visualize proportions (Montessori 2018, pp. 99, 100, 104)



thus allowing us to reaffirm the role of representation in the mathematician's education, and *vice-versa*. Origami models have therefore proved to be tangible models for a prescriptive mathematical modeling as they combine geometric and visuo-spatial thinking. Moreover, origami confirms its powerful meaning as a cognitive artifact which stimulates learning through its own possible interpretation and assimilation

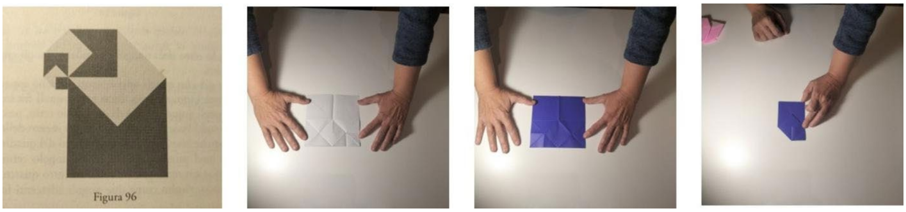


Fig. 4 Proportions and geometric nexus between squares (Montessori 2018, p. 106) and origami folding sequence (author)

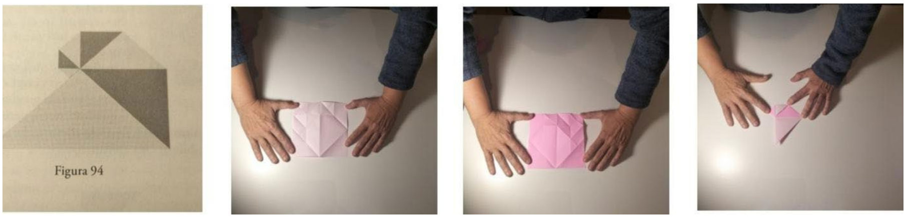


Fig. 5 Proportions and geometric nexus between triangles (Montessori 2018, p. 105) and origami folding sequence (author)

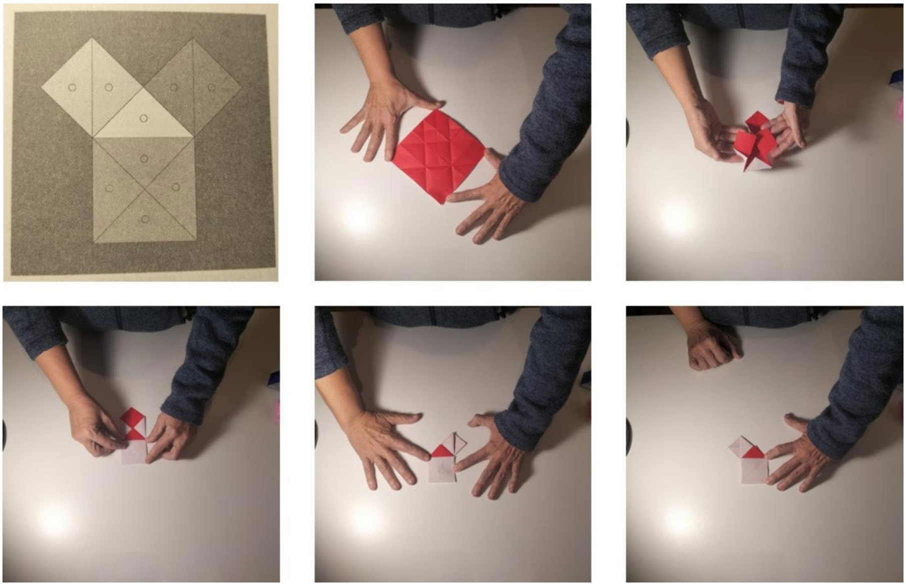


Fig. 6 Pythagorean Theorem (Montessori 2018, p. 206) and origami folding sequence (author)

of the modeling process. Lastly, the origami needs to be designed, requiring users to 'think spatially' and to use the tools of descriptive geometry.

Funding Open access funding provided by Politecnico di Torino within the CRUI-CARE Agreement.

Declarations

Conflict of Interest I declare that I haven't any conflict of interest in my paper.

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