



# Quantifying Urban Network Transitions with Evolution Degree

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## Abstract

This paper proposes a graph-based mathematical approach with a novel metric, the Structure Evolution Degree, to quantitatively analyze urban spatial structure transitions at cross scale and verifies its effectiveness through two cases studies, the San Bartolomeo Quarter of Venice and the Xiaoxihu Historical Block of Nanjing.

**Keywords** Structure Evolution Degree · Transitional morphology · Urban morphology · Urban spatial structures · Graph sheory

## Introduction

In this paper, a new mathematical approach utilizing graphs and a novel metric, the Structure Evolution Degree, is presented to quantitatively analyze the transitions of urban spatial structure at cross scale. Existing graph-based research typically relies on statistical network metrics that fail to comparing graph structures and explaining structural changes in cities. This paper presents a literature review of graph-based research from the perspective of cross-scale mapping and spatial evolution description. The research consists of two main aspects: drawing cross-scale topological graph based on morphological maps and describing the degree of the spatial network evolution. By comparing the spatial structure graphs of the site in different periods, we can observe the permanence and variance of the urban form during the historical evolution. The San Bartolomeo Quarter of Venice and

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the Xiaoxihu Historical Block of Nanjing are used as case studies to verify the effectiveness of this method.

## **Research**

### **Site Information**

#### **Quarter of San Bartolomeo**

The San Bartolomeo Quarter of Venice is studied by Saverio Muratori in his book “Studi per una operante storia urbana di Venezia”. It has four periods of distinct morphological transformation: construction of the parish began in the 11th century; late Byzantine settlements consolidated and expanded in the north of the district in the 14th century; medieval buildings were replaced by masonry in the 18th century, and the urban forms stabilized in the 1950s. Paolo Maretto, Sylvia Tagliazucchi, and Stephen Marshall have studied this district from the perspectives of architectural typology, operative typological approaches by Muratori, and the integration of morphological research and design, respectively.

#### **Xiaoxihu Historical Block**

The Xiaoxihu Historical Block is one of the 22 historic districts in Nanjing China, which has largely preserved the traditional urban fabrics of the Ming and Qing dynasties. It has undergone three typical periods of morphological transformation during the Republican period (the 1930s), after liberation (the 1950s), and in recent years (the 2010s). Since 2016, Southeast University has been conducting a detailed typo-morphological study of the Xiaoxihu. Bao Li, Han Dongqing, Marco Trisciuglio, and Dong Yanan study the block respectively from the perspectives of house diverse renewal, historic district preservation, morphological typology practice and the property plots. Based on these studies, they carried out a small-scale progressive old building renovation.

## **Literature Review**

### **The Topological Networks**

Different topological network graphs describe different mapping methods. Much of the recent research on topological networks has focused on the applications of justified plan graph(jpg) and the analysis of inequivalent genotypes. Lee et al. used directed weighted graphs to draw jpg more precisely and reveal the relationship between visitors and residents in Palladian villas through inequivalent genotypes (Lee, Ostwald, and Dawes 2022). Lucia Elizondo drew jpgs before and after the spatial transformations of Mexican social housing, and compared inequivalent

genotypes of two statuses to show how space configurations are transformed (Elizondo 2022).

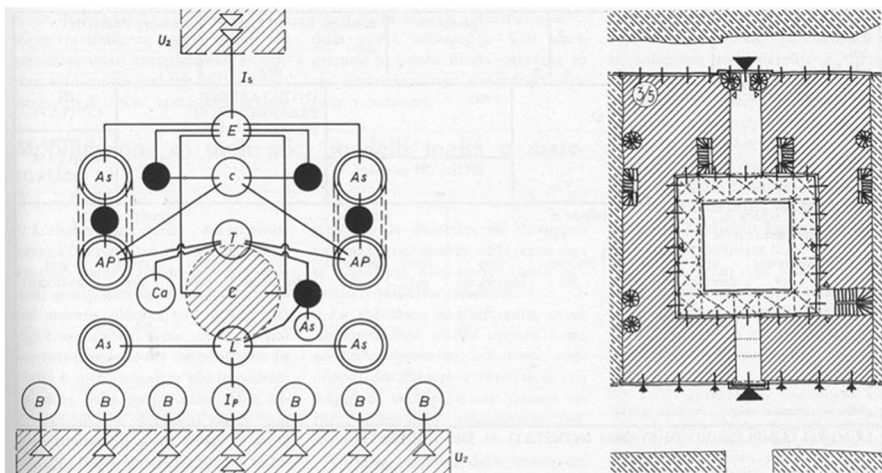
However, most of the existing studies have some limitations: most of analyses are based on the statistical indicators; different scales of studies use different indicators and methods, and so far there is no unified approach across scales.

### Cross-Scale Space Mapping

The key to structuring a cross-scale spatial network is how the transitional space bridges the interior and exterior. Previous studies on transitional spaces include Cavallari Murat's study of Turin morphology, which defines the space enclosed by the doors of Baroque houses as a transitional space linking the urban space and the interior atriums (as shown in Fig. 1). Oscar Newman, in his study of defensible space, extensively used a cross-scale approach, dividing the space into four hierarchies - public, semi-public, semi-private and private (Fig. 2). Similarly, Alexander proposed the transitional pattern of entrance space (pattern 130) and the design pattern of spatial transition hierarchy (pattern 127) in *A Pattern Language* (Fig. 3).

### Space Transitions Description

Urban forms change over time. Most of morphology studies focus on morphology's beginning and final status. However, studying the morphology dynamics of all phases between the beginning and end states can better contribute to the future urban design and operative urban studies (Trisciuglio et al., 2021). Saverio Muratori was an early scholar of the dynamics of urban forms, examining four typical periods of morphology of Venice. Gianfranco Caniggia analyzed the



**Fig. 1** The topological diagrams by Augusto Cavallari Murat. Source: Augusto Cavallari Murat, *Forma Urbana ed Architettura nella Torino Barocca*

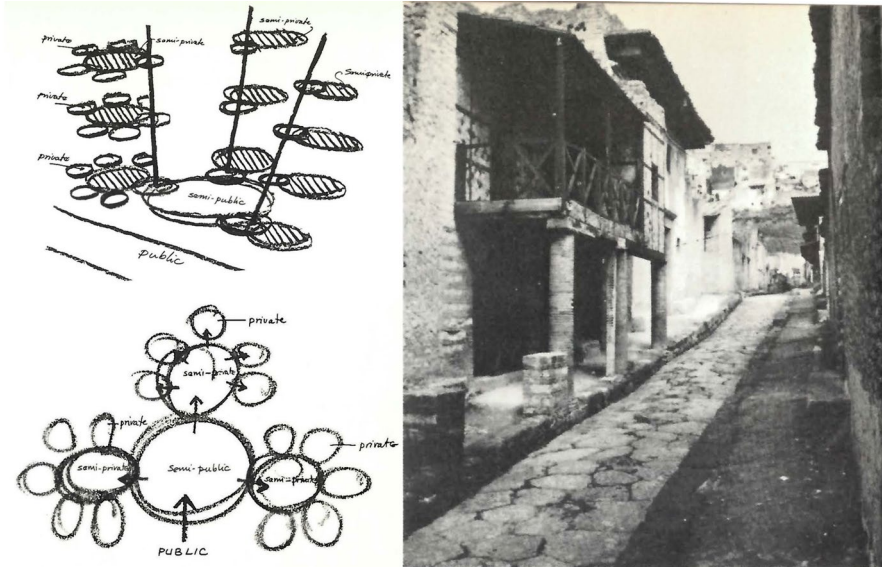


Fig. 2 The hierarchy of defensible spaces and the stoop space in front of the traditional Pompeii house. Source: Oscar Newman, *Defensible space: people and design in the violent city*

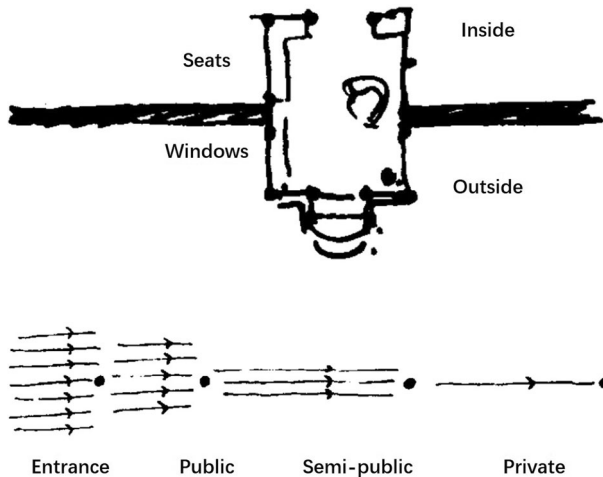


Fig. 3 The pattern 130 Entrance Room and Pattern 127 Intimacy Gradient. Source: Christopher Alexander, *A Pattern Language*

past periods of Como in a stratigraphic overlay, thus providing the developmental premise for planning and designing the future Como (Trisciuglio et al., 2017). The Transitional Morphology Joint Research Unit, founded by the Politecnico di Torino (Italy) and Southeast University (China), has also conducted many studies

on urban morphology transitions, including the evolutionary renewal of Xiaoxihu Block and decoding the gentrification process in Istanbul.

## Mapping Cross-Scale Space

In this study, the spatial network is divided into four levels: public space, front space, courtyard space, and function room space. On the way out, people first walk through the “door” from the semi-private space (such as the indoor living room/corridor/hall) to the “front space” (semi-public space) and then enter the coherent and unified public space. The front space, a special area with territorial significance, is defined as a separate level because the connection between the front space and the street is the most basic spatial structure. It indicates the property relationship of the plot, which usually changes very slowly. For example, if there are ten houses on a street and ten front spaces, they rarely become eleven. A land is usually traded as a whole property plot.

The cross-scale spatial networks consist of spaces (nodes) and spatial relationships (edges). The four colored nodes represent four spatial levels. Black nodes represent complete public spaces such as streets. Dark gray ones are front spaces. Light gray nodes indicate open spaces such as courtyards. White nodes mean functional rooms (Fig. 4). The following drawings take Xiaoxihu Block as an example, and the study of Venice is in progress.

## Description of the Space Structure Transitions

The key to describing the historical changes in spatial networks is to find a mathematical method that allows comparing the structure of topological graphs. Previous network science has focused on statistical studies of the degree distribution of large scale graphs (depth, connectivity, centrality, agglomeration coefficients, degree correlation, etc.), while comparative analysis of small scale graph structures is lacking. In the field of graph theory, several methods are commonly used to compare graphs: graph kernel, graph limit, adjacency matrix comparison, and so on.

Here, for the particular problem of describing the evolution of urban spatial structure, we create a new mathematical analysis index: the Structure Evolution Degree.

A simple example is given to illustrate how this metric is calculated. A simple example is given to illustrate how this metric is calculated. Figure 5 shows the two graphs to be compared. The old graph on the left Graph has evolved to become the new graph on the right Graph'. In the calculation of the weighted evolution degree, the left graph is used to calculate the weighted value ( $V_w$ ) of each node, while the right graph is used to calculate the change rate ( $R_E$ ) of each node.

Rule 1: Structure Evolution Degree (DE) is the weighted rate of change. It is equal to the sum of the product of the weighted value VW and the rate of change RE for each node.

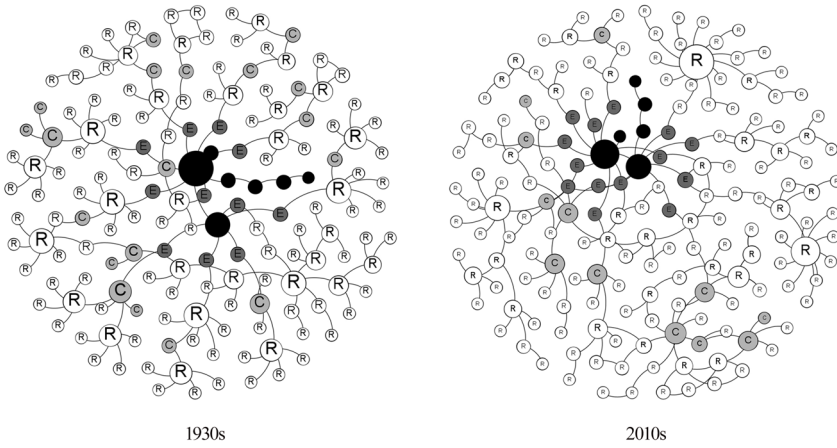


Fig. 4 The transition of Topological Networks of Xiaoxihu, Source: by Author

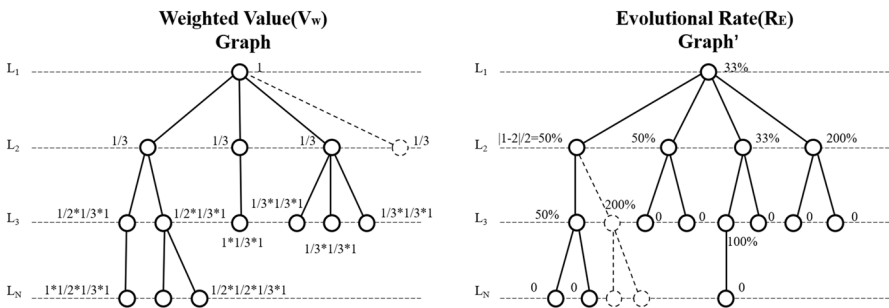


Fig. 5 The case of graphs used to explain DE calculation method, Source: by Author

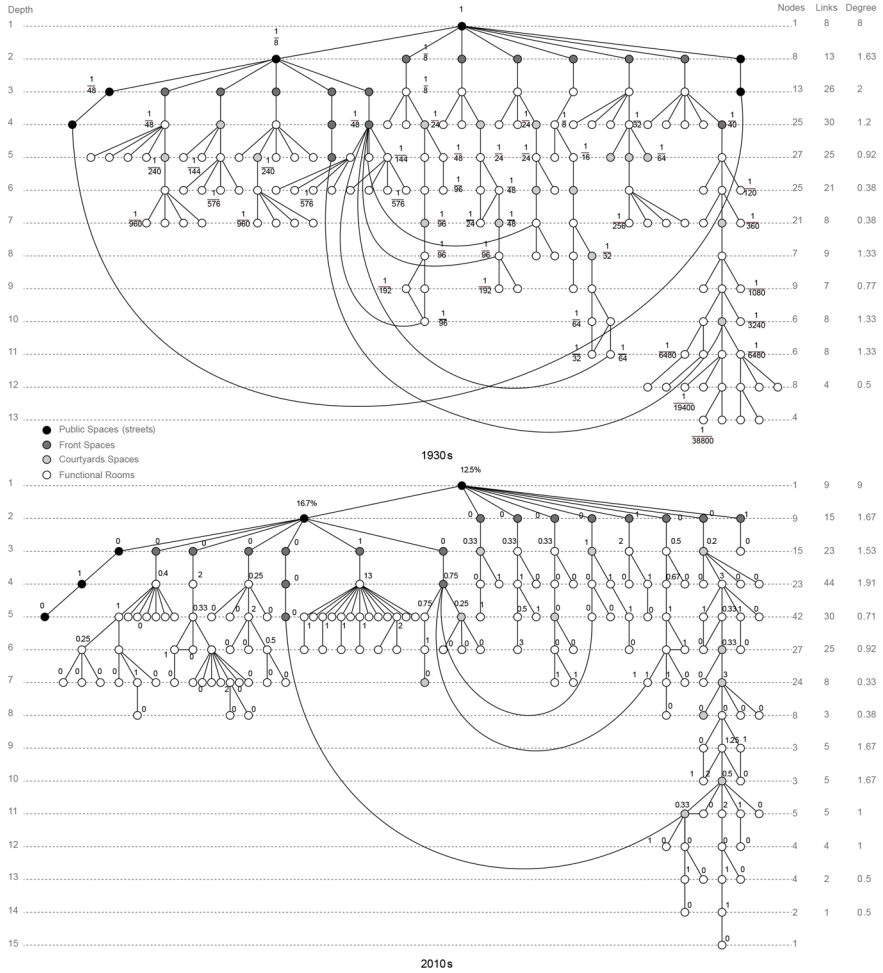


Fig. 6 Calculation Graphs of Xiaoxihu Block in 1930s and 2010 s, Source: by Author

$$D_E = \sum (V_W \times R_E) \quad (1)$$

Rule 2: The weighted value  $V_W$  represents the weight of each node. All child nodes under the same node have the same weight, which is equal to the product of the reciprocal of the total number of child nodes  $N_S$ , the weight of the parent node and the number of edges  $N_E$  on that child node. It is worth noting that the weighted value of the new child node added in  $G'$  takes the value of the child node of the same parent node in  $G$ . So the Weighted Value of node  $i$  located in the  $L_N$  layer (the parent node is node  $j$  located in the  $L_{N-1}$  layer) can be expressed as:

$$V_W(i) = \frac{1}{N_S} \times V_W(j) \times N_E \quad (2)$$

Rule 3: The Change Rate  $R_E$  is a percentage in the range of 0–100%, which is used to describe the increase or decrease of nodes, thus reflecting the change of spatial structure.  $R_E$  means the difference of the total number of children nodes  $N_S(i')$  of a node  $i'$  in the new graph  $G'$  and the total number of children  $N_S(i)$  of node  $i$  at the same position in the old graph  $G$ . So:

$$R_E = \frac{|N_S(i') - N_S(i)|}{N_S(i)} \times 100\% \quad (3)$$

There are two special cases: the addition or disappearance of nodes, and the appearance of a loop connection in the network.

It is specified that when a new child node appears in the new graph, the rate of change of the parent node  $i'$  of that child node is defined as 100% (if two new nodes appear at the same time, the rate of change of node  $i'$  is 200%, and so on). When a child node disappears in the new graph, the rate of change of  $i'$  is also defined as 100% (if two new child nodes appear at the same time, the rate of change of  $i'$  is 200%, and so on).

It is specified that when a loop connection appears in the spatial network, the rate of change of the two nodes it connects is defined as 100% (if two loop connections appear at the same time at the same node, the rate of change of that node is 200%, and so on).

Therefore, the Structure Evolution Degree of the graph in Fig. 5 is:

$$\begin{aligned} D_E &= 1 \times 33\% + \frac{1}{3} \times 50\% \times 2 + \frac{1}{3} \times 33\% + \frac{1}{3} \times 200\% + \frac{1}{2} \\ &\times \frac{1}{3} \times 50\% + \frac{1}{2} \times \frac{1}{3} \times 200\% + \frac{1}{3} \times \frac{1}{3} \times 100\% = 1.96 \end{aligned}$$

After that, we applied this calculation method to a real urban neighborhood case. At present, we have only completed the calculation for the Xiaoxihu Block (Fig. 6).

According to the above three formulas, the Structure Evolution Degree of the Xiaoxihu Block can be calculated as 1.603. (The calculation process is too lengthy to list.)

## Conclusion

This paper shows the application of Structure Evolution Degree to the quantitative assessment of urban spatial structure evolution. Compared with the established studies based on statistical metrics, this method has several advantages as follows:



1. The Weighted Value Graph can correctly reflect that the influence of nodes at different levels on the spatial structure is different. The higher the hierarchy of nodes, the more basic the spatial structure is reflected.
2. The Evolution Rate Graph can describe very accurately how much the graph has changed. It is the result based on comparing the difference between the two graphs.

This method is still in the process of development. Structure Evolution Degree is heavily affected by the size and depth of the graph. It is difficult to directly compare the Structure Evolution Degree among different size communities. Some of the current algorithm rules for specific cases are not very reasonable. In the future, we will combine more complex mathematics to improve the calculation formulas.

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## Declarations

**Conflict of interest** On behalf of all authors, the corresponding author states that there is no conflict of interest.

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