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CONFERENCE PAPER

Complexity and Diversity in Palladian Facades

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Abstract
This study presents an analysis of two aesthetic properties, *complexity* and *diversity*, in Palladian architecture. The former highlights the fractal dimension of façade geometry, and the latter its semantic randomness. Along with a methodological description and advice on settings, this study contributes to the discussion about mathematical beauty in architecture.

Keywords Fractal Dimension · Complexity · Information Theory · Diversity · Perplexity · Palladio · Aesthetics · Design analysis · Palladian architecture · Fractals · Randomness

Introduction

Due to its geometric purity and consistency, Palladian architecture has been frequently explored using computational methods, most of which are focused on analysis and generation of plans (Lee and Ostwald 2020). Two exceptions are García-Salgado's (2008) and Fletcher's (2015) studies of the proportional characteristics of the *Villa Rotonda*'s and the *Villa Emo*'s facades, respectively. Eilouti (2008) also describes and generates Palladian façades using a formal language derived from a finite set of design elements (vocabulary). Other than these proportional and grammatical studies of Palladian elevations, few alternative methods have been used to examine their aesthetic properties. This research addresses this knowledge gap using two computational methods to measure aesthetic complexity and diversity in Palladian architec-

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ture. Across a series of applications of these methods to Palladian facades, this paper develops new mathematical readings of their aesthetic properties and compares two methods.

The Research

A common argument is that the beauty of Palladian architecture arises from a combination of its proportions, symmetry, rhythm, and harmony. Furthermore, in terms of *visual stimuli*, complexity and diversity have been identified in past research as strong determinants of architectural aesthetics (Stamps 2003; Lee and Ostwald 2023). As such, the present research employs two mathematical measures – complexity is 'the state of being complicated', while diversity is 'the state of being diverse' – to examine the visual character of Palladian architecture. This research adopts fractal dimension (*complexity*) and perplexity (*diversity*) measurements to calculate the aesthetic values.

Complexity

To statistically measure complexity in an image or object, fractal dimension analysis has been adopted in various domains including architecture (Lee and Ostwald 2023). Specifically, fractal dimension, "the relative density of geometric information" (Ostwald and Vaughan 2016, p. 3), is used to measure and compare '*architectural aesthetic character*'. Likewise, as a significant predictor of beauty, it is used to quantify visual complexity in architecture (Lee and Ostwald 2023). Traditionally, the box counting method, a measure of the distribution of data in a set, has been widely used for this purpose. The fractal dimension (D) of an architectural façade image is:

$$D = \lim_{\epsilon \to 0} \frac{\log N_s}{\log(1/s)} \tag{1}$$

where N_s is box count and s is box size. The final D is the slope of the trendline for the log-log plot of N_s versus s. In this way, the D values of architecture are between 1 and 2, indicating the degree of complexity (Ostwald and Vaughan 2016). That is, a lower D indicates a less complex design, while a higher D, a more complex one. Past studies have identified different D ranges in a variety of architectural data. For example, the interquartile D range of the elevations of 85 canonical domestic buildings (designed or built in between 1901 and 2007) is from 1.3698 to 1.5104 (Ostwald and Vaughan 2016). Likewise, knowledge about the D range of a certain style of architecture can be used to examine the visual complexity of a design within such a style.

To develop D measures, image processing and setting parameters such as grid sizes and positions must be employed in a consistent way (Ostwald and Vaughan 2016; Lee and Ostwald 2023). For the present research, each façade is developed from the original elevation published in *the Quattro libri'* (Palladio 1570), although statues and their pedestals are not considered in this analysis. In addition, window and door frames are also simplified as a single line as illustrated in Fig. 1.

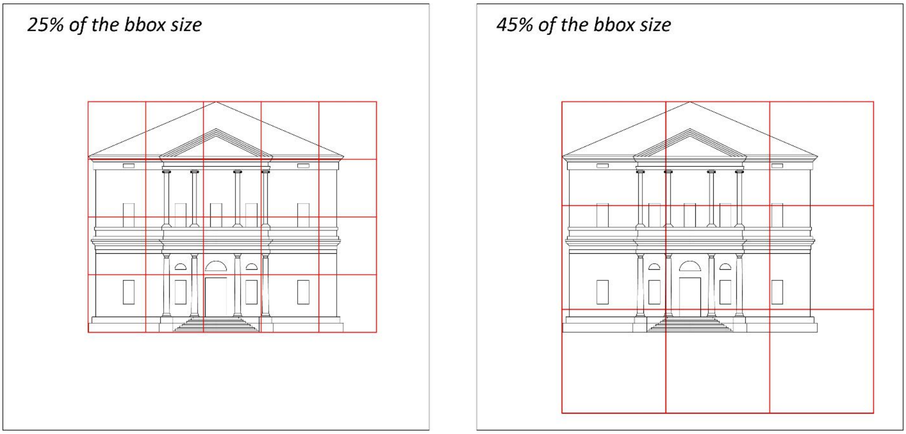

Fig. 1 *Villa Pisani*, façades with two maximum grids (25% and 45%) from a top-left corner position of the *bbox*

Table 1 The box counting options for the fractal dimension analysis

	Option	*Description*
Maximum grid size	25%	25% of the *bbox* size (similar to 0.25 *L* where *l* is the length of the shorter side of the *bbox*)
	45%	45% of the *bbox* size (default option suggested by *FracLac*)
Grid position	Four corners	Four corners of the *bbox*
	12 positions	After four corner and randomly added orientations around the initial four locations by *FracLac*
Scaling coefficient	√2 (or 1.414)	√2 :1 (approximately, 1.414:1)
	ϕ (or 1.618)	Golden ratio ϕ (approximately, 1.618)

Using *ImageJ* with its plug-in (*FracLac*) for the fractal dimension analysis, the maximum size of a bounding box (*bbox*) – the smallest rectangle enclosing the pixelated part of an image – considers a resolution of 4096×4096 ($2^{12}\times2^{12}$) pixels and the minimum grid size in Fig. 1 is 10 pixels. In contrast, this study tests several options for the box counting method in Table 1 because the *D* values may be dependent on the settings. Specifically, the golden ratio (ϕ) is used for one 'scaling coefficient' option because Palladian facades can be interpreted using it (Fletcher 2015).

Diversity

Diversity is "the quality or state of being composed of many different elements or types" (Merriam-Webster n.d.-b). Complexity is the state of having many different parts interrelated in a complicated way, while diversity is focused on the fact that a system contains many different elements. Avoiding 'monotony' and 'chaos', visual diversity is regarded as an *architectural aesthetic variable* (Stamps 2003). In particular, the diversity or richness of a façade can be measured by its degree of randomness or visual entropy. That is, Shannon's (1948) information entropy can be used for a mathematical formulation of visual diversity in a facade (Stamps 2003). The information entropy (*H*) is:

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i \tag{2}$$

where p_i is the probability of observing a specific event *i*. In architecture, the probability can be calculated by the frequency of a design component or element (Stamps 2003). As such, the *H* value can be regarded as the total volume of all possible information of identified design elements in a façade, highlighting a measure of '*uncertainty*'. In contrast, the property of "perplexity", the exponential of *H*, is also used as a diversity index (Jost 2006). The perplexity (*PP*) of a probability distribution is:

$$PP = \prod_{i=1}^{n} p_i^{-p_i} \tag{3}$$

Theoretically, the perplexity of a probability distribution might be a better index for the diversity of a system, but the entropy itself is also a reasonable index of diversity (Jost 2006). Thus, the present research calculates both *H* and *PP* of Palladian architecture. Counting the frequency of a design element is essential for these diversity measures. Following Eilouti's (2008) language, the present research develops the vocabulary set for Palladian facades in Table 2. As illustrated in Fig. 2, design elements in a façade are encoded and the probability of each code in the information string is used to calculate the diversity indexes (*H* and *PP*).

Table 2 The vocabulary set for Palladian facades

Category	*Code*	*Description*
Pediment, Roof	PE, RO	Pediment, roof, respectively
Entablature	E1	Entablature
	E2	Entablature without architrave
	E3	Entablature with window
	E4	Entablature with pedestal
Wall	W1	Wall without openings
	W2	Wall with window
	W3	Wall with two windows
	W4	Wall with door
	W5	Wall with door and window
Column	C1 to C4	Doric, Ionic, Corinthian, Composite, respectively
Stair	S1, S2, S3	Stair, stair with door, stair post, respectively
Pedestal	P1	Pedestal
	P2	Pedestal with window

Fig. 2 *Villa Emo*, a façade and design elements with codes

Pilot Study

This study examines the complexity and diversity of three Palladia façades, *Villa Emo*, *Villa Saraceno* and *Villa Pisani*. As an example, the information string of design elements in the *Villa Emo* (Fig. 2) is:

RO PE E2 E1 E2 W2 C1 W2 C1 W4 C1 W2 C1 W2 P2 P1 S3 S1 S3 P1 P2.

Thus, the *PP* value of this façade is:

$$PP = p_{Ro}^{-p_{RO}} \times p_{PE}^{-p_{PE}} \times p_{E2}^{-p_{E2}} \times p_{E1}^{-p_{E1}} \times p_{W2}^{-p_{W2}} \times p_{C1}^{-p_{C1}} \times p_{W4}^{-p_{W4}} \times p_{P2}^{-p_{P2}} \times p_{P1}^{-p_{P1}} \times p_{S3}^{-p_{S3}} \times p_{s1}^{-p_{s1}}$$

$$= \frac{1}{21}^{?\frac{1}{21}} \times \frac{1}{21}^{?\frac{1}{21}} \times \frac{2}{21}^{?\frac{2}{21}} \times \frac{1}{21}^{?\frac{1}{21}} \times \frac{4}{21}^{?\frac{4}{21}} \times \frac{4}{21}^{?\frac{4}{21}} \times \frac{1}{21}^{?\frac{1}{21}} \times \frac{2}{21}^{?\frac{2}{21}} \times \frac{2}{21}^{?\frac{2}{21}} \times \frac{1}{21}^{?\frac{1}{21}} \times \frac{1}{21}^{?\frac{1}{21}} = 9.5101$$

In this way, the entropy (*H*) values of the three villas are 3.2495, 3.4399, and 3.3927, and their diversity (*PP*) values are 9.5101, 10.8524, and 10.5028. Although the *Villa Saraceno* looks like the *Villa Emo*, being a one-storey building, it has the highest degree of *diversity*. In contrast, the *Villa Pisani*, has the largest number of design elements, but only the second level of diversity because many parts are repetitive. These results arise from the fact that diversity measures in architecture deal with sematic design elements rather than geometries.

This study also tests box counting options for the complexity measure. Figure 3 describes the *D* results of the Palladia façades using these options. Regardless of the options, all results in the figure suggest that the *Villa Pisani* has the highest level of complexity, and the *Villa Emo* has the least level of complexity. However, when considering the standard errors of *D* results, the optimal settings are:

- the maximum grid size being 25% of the *bbox* size,
- 12 grid positions including four corners of the *bbox*, and.
- the golden ratio (ϕ) for scaling coefficient.

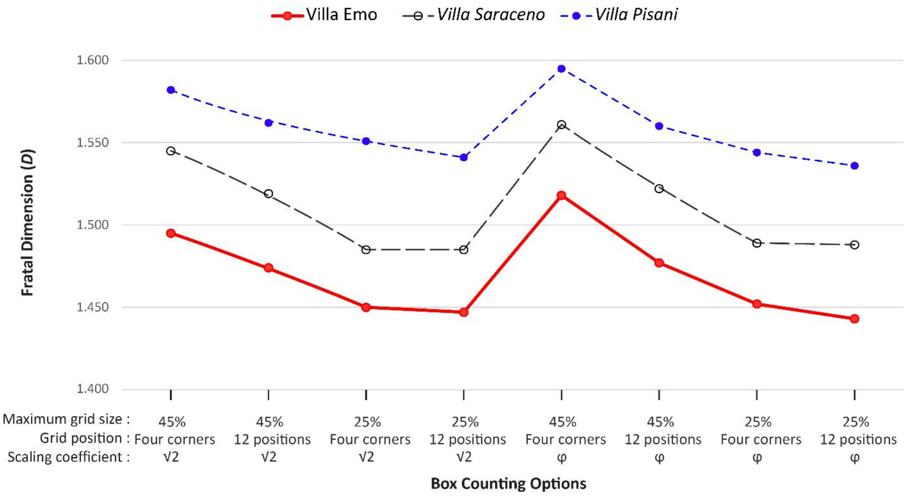

Fig. 3 The *D* results of three Palladian façades by different counting options

Conclusion

This research reports the results of a comparison of two aesthetic properties of Palladian architecture – complexity and diversity. The twin methods provide an opportunity to understand the aesthetic character of Palladio's façades, as they relate to visual preferences and perceptions. Conceptually, complexity (*D*) is more like a geometric snapshot of a whole, while diversity (*H* or *PP*) is more like a sum of sematic parts. It might be intuitively assumed that a relationship exists between complexity and diversity, "more diversity, more complexity", however, the results of this study may better reflect a Gestalt principle, "*the whole is other than the sum of the parts*". Complexity may also be impacted by the strict axial symmetry of Palladian architecture. A future study will further test these hypotheses, providing new, mathematical insights into the character of Palladian architecture.

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