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Bernardo Vittone's room height method and the golden ratio

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Abstract
This paper analyzes Bernardo Vittone's self-invented room height method by translating it algebraically and comparing it with the arithmetic, geometric and harmonic means. The conclusion suggests that Vittone might have used the golden ratio to develop his room height method.

Keywords Bernardo Vittone · Design theory · Geometric analysis · Architectural proportions · Golden ratio

In this paper, Bernardo Vittone's multifarious theory of proportion is discussed by concentrating on a particular example from his treatise *Istruzioni elementari*. Despite his repeated emphasis on the importance of harmonic proportions throughout his treatises, Vittone's most important architectural works are predominantly composed by way of geometric operations. Generally, it makes little sense to look for harmonic proportions in his church designs, for which he is most famous. The relationship between numeric and geometric proportions in his architectural works and writings is therefore somewhat ambiguous. In the third book of the *Istruzioni elementari*, after having considered Vincenzo Scamozzi's and Andrea Palladio's room height methods, Vittone offers one of his own as an alternative. Vittone's self-invented method entails a geometrically determined mean function of length and width. First, this paper provides an algebraic translation of Vittone's room height method by way of the cosine rule. Second, it compares its practical application with the arithmetic, geometric and harmonic means. The outcome is a speculation on the possibility that Vittone might have used the golden ratio to develop his idiosyncratic room height method.

Vittone's room height method pertains primarily to *sale*, that is, grand, representative rooms in houses or palaces. According to him, length–width ratios

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in quadrangular rooms should be ‘harmonic’, and the length should never exceed twice the width (Vittone 1760: 450–451). Accordingly, he recommends the ratios 1:1, 1:2, 2:3, 3:4, 4:5 and 3:5, all of which are musical or ‘harmonic’. Regarding room height, he first recommends Scamozzi’s method of employing the arithmetic mean of length and width to determine the height of quadrangular rooms (Scamozzi 1615: 306–310). He then introduces the geometric mean, which he attributes to Palladio, and notes that Palladio also recommended the height of square rooms to be 4/3 of the width (Palladio 1570: 53–54). The fact that Palladio also prescribed the arithmetic and harmonic means goes unmentioned. Vittone deems both Scamozzi’s and Palladio’s methods for square rooms unsatisfactory; the former makes rooms too low and the latter too high (Vittone 1760: 451). “In order to proceed in this [matter] in a regular way”, he therefore continues, “one can observe the following method for [determining the height of] any kind of room” (Vittone 1760: 451, my Eng. trans.).

The self-invented method that Vittone subsequently presents is strictly geometrical and, accordingly, it is at odds with the common notion of *symmetria* that is stressed throughout the entire Vitruvian tradition, including the *Istruzioni elementari* (Vittone 1760: 237–238). Vittone’s room height method, which is supplied by a diagram (see Fig. 1), goes as follows:

> “With the interval AB equal to the width of the room, one shall draw the semicircle BCD. And this [semicircle] shall be divided into two parts BC and CD so that BC has to CD the same [ratio] as the room’s length has to its width. The same shall be the case with the radius AB, by dividing it in the point E so that the portion AE has to EB the same proportion as the room’s length has to its width. That being done, one shall draw from the point E to the point C, a straight line EC. And this shall be the height assigned to the room.” (Vittone 1760: 451, my Eng. trans.)

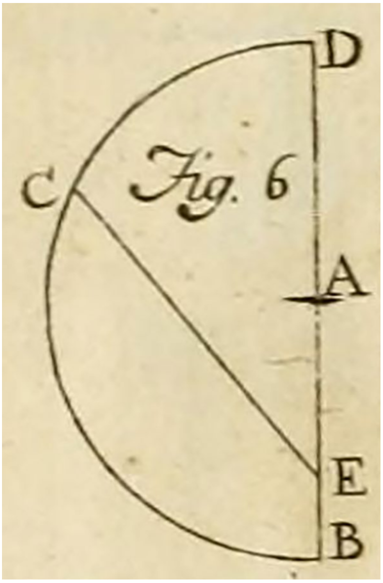

Fig. 1 Vittone’s diagram in the *Istruzioni elementari*, vol. 2, plate 78.

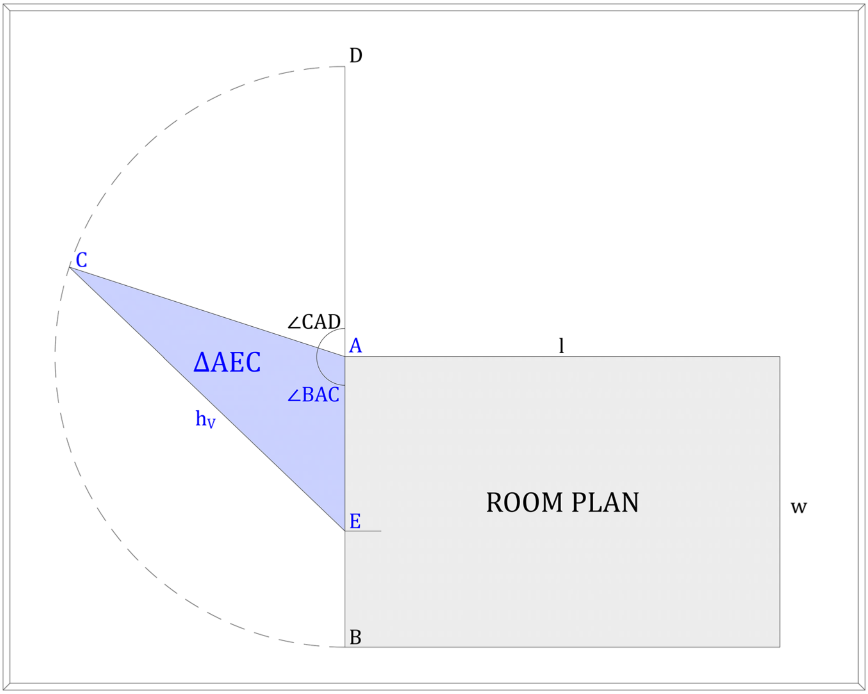

Fig. 2 Modified diagram showing room plan in relation to the semicircle in which the obtuse triangle ΔAEC is drawn.

The procedure encompasses that room height is a geometric function of length and width (see Fig. 2). The room width is set as the radius of a semicircle (*arc BD*) which, in turn, is divided in two by the point C. The major *arc BC* and the minor *arc CD* must have the same proportional relationship as the room's length and width, respectively. Accordingly, this proportional relationship applies to the angles $\angle BAC$ and $\angle CAD$ within the sectors $\diamond BAC$ and $\diamond CAD$, defined by the center point A and the two aforementioned arcs. The same proportional relationship must be employed when dividing the room width (or line AB) itself into two parts separated by the point E. Finally, the line between the points C and E constitutes the height (from now on called h_V) of the room. The proportional relationships on which Vittone's room height method is based may be summarized in the following manner:

$$\frac{l}{w} = \frac{AE}{EB} = \frac{\angle BAC}{\angle CAD}$$

If one applies the cosine rule, Vittone's method for determining room height h_V may be expressed algebraically as the unknown side length of the obtuse triangle ΔAEC which is defined by the known angle $\angle BAC$ and the two known side lengths AE and CA (see Fig. 2). It is therefore necessary to express $\angle BAC$, AE and CA by means of the room's length l and width w. Since

$$\frac{\angle BAC}{\angle CAD} = \frac{l}{w}$$

and

$$\angle CAD = 180 - \angle BAC$$

the angle $\angle BAC$ in the obtuse triangle ΔAEC may be expressed as:

$$\angle BAC = \frac{l}{w}(180 - \angle BAC) = \frac{180l}{w} - \frac{\angle BACl}{w}$$

$$\angle BACw = 180l - \angle BACl$$

$$\angle BACw + \angle BACl = 180l$$

$$\angle BAC(w + l) = 180l$$

$$\angle BAC = \frac{180l}{l + w}$$

The side AE in the obtuse triangle ΔAEC may be expressed as follows: Since.

$$\frac{AE}{EB} = \frac{l}{w}$$

EB can be expressed as:

$$EB = \frac{AEw}{l}$$

Hence, we have that.

$$AE = w - EB = w - \frac{AEw}{l}$$

and furthermore, by dividing by AE, we have that

$$1 = \frac{w}{AE} - \frac{w}{l}$$

$$\frac{w}{AE} = 1 + \frac{w}{l}$$

$$w = \left(1 + \frac{w}{l}\right)AE$$

Hence, AE may be expressed as:

$$AE = \frac{w}{1 + \frac{w}{l}} = \frac{lw}{l + w}$$

The other known side of the obtuse triangle ΔAEC is CA. Since CA is the radius of the semicircle, it is equal to the room width w. Therefore, the following may be summarized:

$$\angle BAC = \frac{180l}{l + w}, AE = \frac{lw}{l + w}, CA = w$$

This provides the necessary information for the obtuse triangle shown in Fig. 3.

For determining the unknown h_V in the obtuse triangle ΔAEC, the cosine rule is applied as follows:

$$h_V{}^2 = CA^2 + AE^2 - 2CA * AE * cos\angle BAC$$

Using substitutes expressed by means of room length l and room width w, the cosine rule equation appears as:

$$h_V{}^2 = w^2 + \left(\frac{lw}{l + w}\right)^2 - 2w\left(\frac{lw}{l + w}\right)cos\left(\frac{180l}{w + l}\right)$$

which means that Vittone's room height mean h_V can be expressed as

$$h_V = \sqrt{w^2 + \left(\frac{lw}{l + w}\right)^2 - 2w\left(\frac{lw}{l + w}\right)cos\left(\frac{180l}{w + l}\right)}$$

If one applies Vittone's room height rule to an imaginary room which is 12 feet wide and whose length is set according to his prescribed ratios (1:1, 1:2, 2:3, 3:4, 4:5 and 3:5), the height h_V may be expressed as

$$h_V = \sqrt{144 + \left(\frac{12l}{12 + l}\right)^2 - 24\left(\frac{12l}{12 + l}\right)cos\left(\frac{180l}{12 + l}\right)}$$

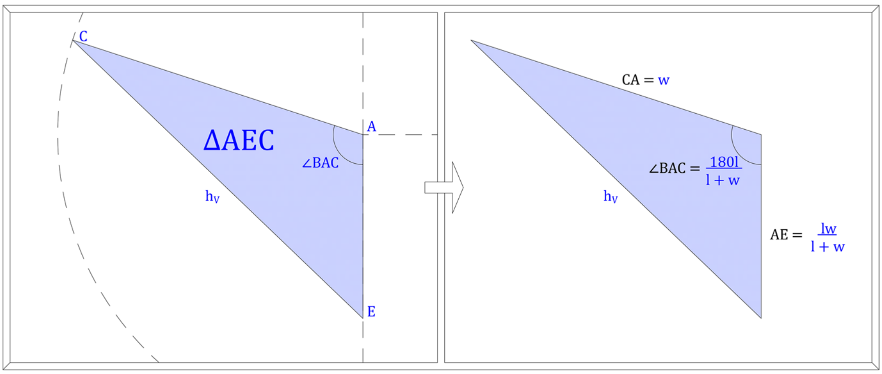

Fig. 3 The obtuse triangle to which the cosine rule may be applied.

Table 1 Vittone's height-mean compared to the arithmetic, geometric and harmonic means

Ratio	w, l	h_a	h_g	h_h	h_V
1:1	12, 12	12	12 [Palladio: 16]	12	13.4164…
4:5	12, 15	$13^{1/2}$	13.4164…	$13^{1/3}$	14.7047…
3:4	12, 16	14	13.8564…	$13^{5/7}$	15.0878…
2:3	12, 18	15	14.6969…	$14^{2/5}$	15.7873…
3:5	12, 20	16	15.4919…	15	16.4053…
1:2	12, 24	18	16.97…	16	17.4356…

It is now possible to compare Vittone's room height mean (h_V) with the arithmetic (h_a), geometric (h_g) and harmonic (h_h) means when applied to the same room proportions (see Table 1).

$$h_a = \frac{w+l}{2}, h_g = \sqrt{wl}, h_h = \frac{2wl}{w+l}$$

As expected, Vittone's room height mean (h_V) results in irrational numbers which, accordingly, stand in an incommensurable proportional relationship with length and width. Vittone's room height mean also produces higher values than the three other means, except for Palladio's principle of employing 4/3 of the width for the height in square rooms and the arithmetic mean employed in double square rooms. Accordingly, as Table 1 shows, Vittone's method is only a middle ground between Scamozzi and Palladio when it comes to double square and square rooms. Although Vittone presented it as applicable to any kind of room, his method seems to be intended for rectangular rooms rather than square ones, for which he could have provided a much simpler rule. Despite this, an indication of how Vittone might have developed his room height method in the first place transpires if one considers its application to square rooms. For square rooms only, it involves the golden ratio (ϕ) in the following way (see Fig. 4 as well):

$$\phi = \frac{h_V + {}^1\!/_2 w}{w}$$

Accordingly, the height in square rooms can be expressed as:

$$h_V = w\left(\phi - {}^1\!/_2\right)$$

Therefore, one may contemplate upon the possibility that experiments with the golden ratio and square rooms could have been the starting point when Vittone invented his room height method. This hypothesis is strengthened by the fact that he presents his room height method immediately after having lamented Scamozzi's and Palladio's room height prescriptions for square rooms. Although he does not promote the golden ratio as a preferable architectural proportion, nor does he cite the seminal Renaissance treatise on the topic, Luca Pacioli's *Divina Proportione*

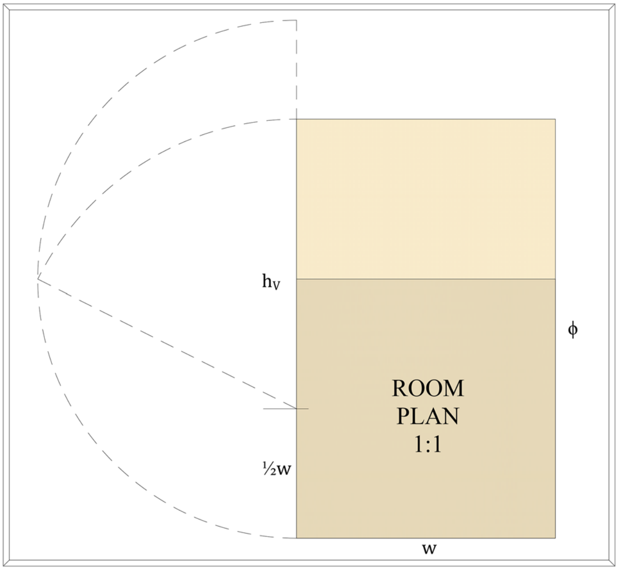

Fig. 4 Vittone's room height method applied to a square room.

of 1509, he did in fact own a copy of it (Portoghesi 1966: 250). This may further nourish our speculation. Moreover, Vittone's treatises are permeated by an interest in esoteric and mystical ideas pertaining to architecture, if not in practice, then at least in theory. Not only did he insist on the importance of harmonic proportions in the *Istruzioni elementari*, which also includes a chapter entitled 'On the generation and nature of musical proportions' (Vittone 1760: 245–251). Also, in the *Istruzioni diverse*, he appended a brief, speculative treatise by his assistant Giovanni Battista Galletto entitled 'Harmonic instructions' (Vittone 1766: 219–324). The *Istruzioni elementari* is dedicated to God (and the *Istruzioni diverse* to Virgin Mary) whom he addresses in the introductory dedication as the sovereign architect of the universe which, according to Vittone, is comprised of shapes only imperfectly perceivable to humans (Vittone 1760: Dedication). In other words, Vittone's tone and inclinations frequently echo authors such as Pacioli.

As pointed out by Hanno-Walter Kruft, however, the golden ratio itself played no significant role in early modern architectural theory except as subject for esoteric speculation (Kruft 2004: 70). Even if the present speculation should bear a hint of truth, it would not change Kruft's point but only, perhaps, slightly and cheerfully modify it.

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Data Availability The author confirms that the sources used in this study are accounted for within the article and its reference list.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interests.

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