



Diagrams of Spatial Experience Based on Mathematical Models

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Abstract

This study focuses on diagrams that use mathematical models to describe the spatial experience, from 1920 to 2020 s. By reviewing and discussing the diagrams classified by the two main mathematical models, topology and encoding, this study tries to rediscover the empirical studies of spatial experience.

Keywords Diagram · Spatial Experience · Topology · Encoding · Design Theory

Introduction

Bringing better spatial experience has been what architects aim to achieve for the past century, while predicting and simulating one's spatial experience accurately is almost an impossible task for those who don't have enough experience. Tools and techniques were invented in the progress to support designers to catch what users might really perceive. Diagrams that reflect the interaction between users and space is one of those tools.

This study focuses on diagrams defined above, to review those typical ones by their mathematical models which play vital roles when describing spatial experience quantitatively. There are at least two types of mathematical models in architectural diagrams - the static and the dynamic, depending on whether the time variable is taken into account, among which *topology* and *encoding* are two of the most important. By reviewing diagrams and their corresponding mathematical models, this study

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attempts to rediscover the empirical studies of spatial experience, which may offer architects evidence for their research and design.

The Research

This paper is expected to answer two questions: (1) How the diagrams and their mathematical models can describe / simulate spatial experience. (2) The causes and main purposes of the diagrams.

Regarding the first question, systematic literature research has been conducted. Then diagrams are carefully categorized based on mathematical models. There are two types mainly: *diagrams based on topology*, and *diagrams based on encoding*. Before going further with each type of diagrams, features that make a diagram capable of describing spatial experience have been paid very little attention by scholars and thus should be discussed at the very beginning, to make a clarification of why diagrams in this paper being selected. First, the diagram should contain a process of abstraction, transforming the tangible form into parameters that can be measured and computed. Second, the diagram should express human perception from subjective perspective, no matter it is observed by architects, or concluded through interviews and questionnaires, or practically tested and measured by wearable sensors. Third, the perception should be reflected within the spatial structure in a spatiotemporal way. The interaction between users and the spatial interface is underlying in the models when trying to quantify it (Fig. 1. Diagram selection, author's own painting).

To answer the second question, the diagrams of each category are juxtaposed and cross-referenced in order to analyze their causes and purposes. By referring to mathematical models adopted, this paper tries to reveal the diagrams' effort in predicting and simulating spatial experience, their characteristics and commonness, and then puts forward the future expectation.

Previous research has highlighted two classic diagrams that enlighten those later ones: Generative Diagram, and Labanotation, which provide *topology* and *encoding* as two ideas of spatial experience expression, respectively. Generative Diagram generates design by implementing logical changes, including *topology*. As a result, Digital Diagram and a number of variants can be considered the continuation of this context (Yang 2019). Correspondingly, Labanotation is thought to be the first to inspire architects to record or simulate people's motion in space by notations (Wei 2020: 70–74) (Fig. 2. The origin of labanotation, Rudolf Laban 1928).

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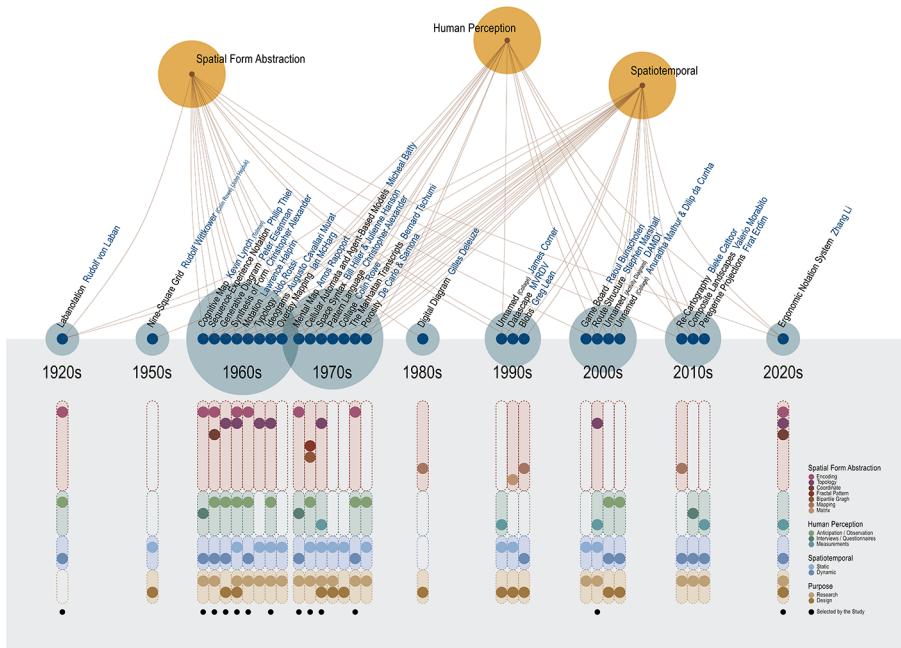


Fig. 1 Diagram selection, from which the diagrams that meet the three conditions are selected in this paper. Author’s own painting.

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Accordingly, the following representative diagrams will be briefly introduced in their respective categories to demonstrate the role of mathematical models in describing spatial experience (Fig. 3. Representative diagrams, author’s own painting).

Diagrams Based on Topology

Euclidean geometry in transforming architectural spaces into more abstract structure is nothing new, especially in morphological study, when trying to simplify a complex into a pattern or a configuration. It could be considered the first step when describing spatial experience to extract the most relevant parameter from jumbled spatial information, i.e. the inner relations between spaces, which is why *topology* is adopted at the first place.

One successful example of simplification is Christopher Alexander’s *Synthesis of Form* (1964), in which the relationship between human’s activities and spaces is subdivided into multi-layers. However, the structure highly relies on architects’ personal experience which makes its logic of aggregation lack of simplicity and replicability.

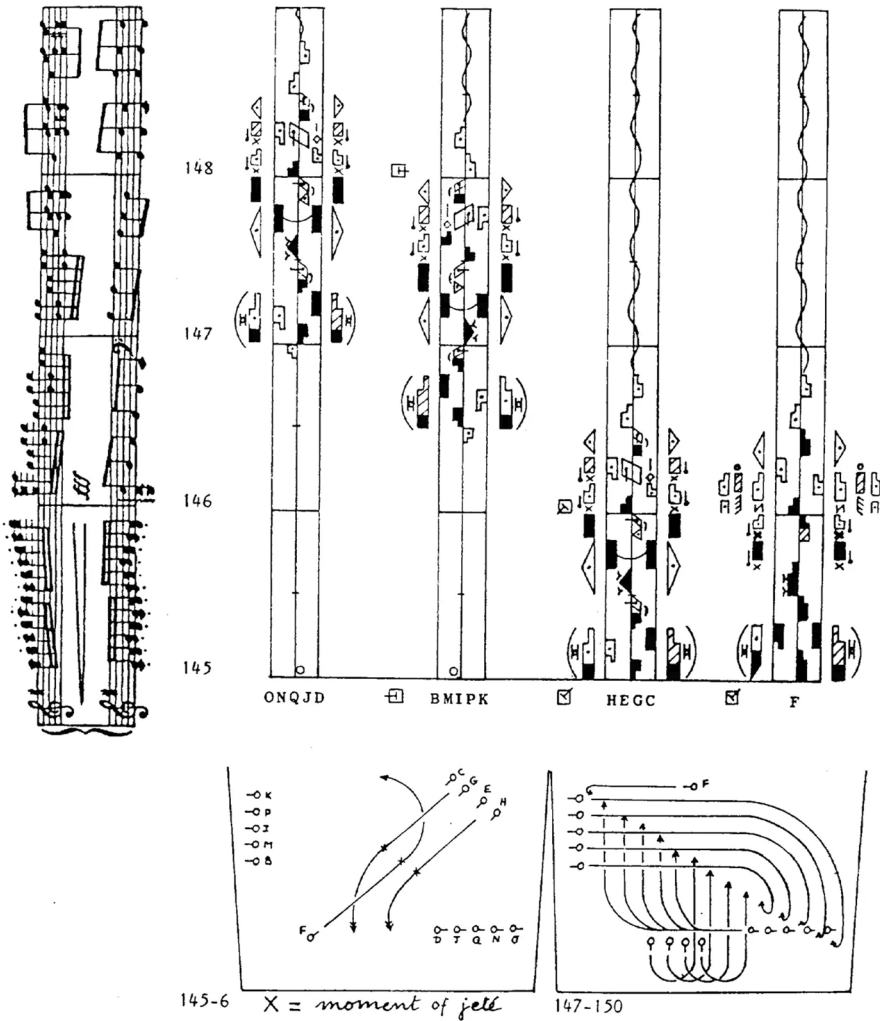


Fig. 2 The origin of labanotation, which is regarded as the first diagram that inspired architects to describe people’s motion in space by notations. A page from Laban’s *Schriftanz* (1928).

Later on, Augusto Cavallari Murat invented the Ideograms to present the connection between public and private space, the hierarchy of connection, and the importance of certain buildings based on a topological network. Stephen Marshall developed Route Structure, a progressive abstraction of network structure from composition (geometry) to configuration (topology) to constitution (hierarchy), to describe the transport experience in a city area.

It’s worth mentioning that Space Syntax, established by Bill Hillier, makes a significant contribution to analyzing space perception by quantifying space configurations (Zhang and Wang 2004: 33–44), rather than simply concluding topological relations like those previous diagrams. Intuitively, Space Syntax describes the space

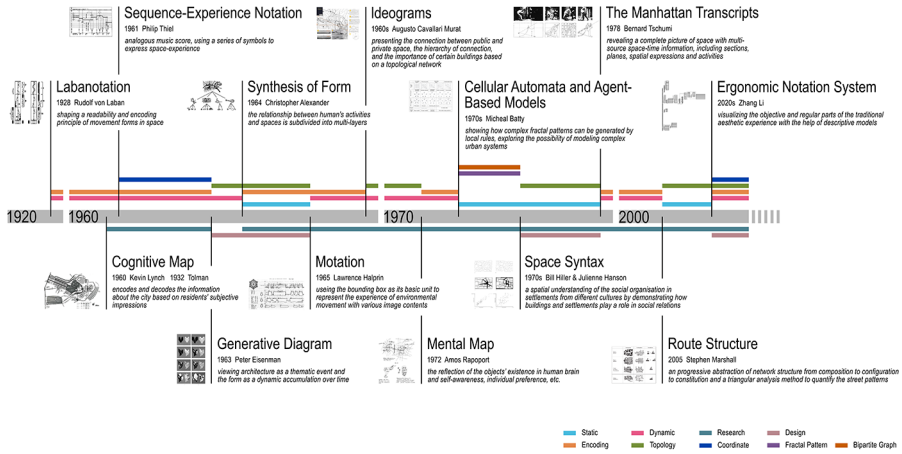


Fig. 3 Representative diagrams, whose main features and mathematical models are briefly introduced in this paper. Author’s own painting.

form using a Justified Graph based on topology, and then creates a series of variables to quantify the description, including connectivity value, control value, depth value, integration value, and intelligibility. Several diagrams, such as Axial Map and Isovist Map, were developed in the following decades to simulate people’s movement and vision on a static city map, as well as to attempt to break the divide between research and design practice. However, some scholars question if a simplified representation of the space network is still necessary in the modern era given that new computer techniques may offer a comprehensive analysis based on a richer support (Ratti 2004: 487–499).

Diagrams Based on Encoding

Compared with *topology*, *encoding* puts more effort in measuring human’s activities and perception than spatial structure, in which the sequence of motion is considered above all while the spatial form usually takes little part. Inspired by Labanotation, Sequence-Experience Notation (Thiel 1961: 33–52) and Motation (Cureton 2017: 104–116) employ a series of symbols analogous to music score to express spatial experience. Later on, Bernard Tschumi established The Manhattan Transcripts to reveal a complete picture of space with its form and what happens in space.

There is a parallel development in which human perception is measured by statistics from the crowd’s *image*. Carried forward by Kevin Lynch, Cognitive Map (1960) encodes and decodes the city image from paths, landmarks, edges, districts and nodes, based on residents’ subjective impressions - ‘imageability’ named by Lynch. After it, Mental Map (Rapoport 2016) describes the ‘imageability’ as the reflection of the objects’ existence in human brain. Most recently, what Lynch and Rapoport created was developed into a computational approach by Filomen et al.

It’s worth noting that Ergonomic Notation System (Zhang et al. 2022: 1744–1756) was recently proposed to quantify the spatial experience using time sequence encod-

ing. Combined with emerging ergonomic measurements, i.e. EDA, EEG, EMG, eye movement, etc., ENS makes visualizing human perception and activities within a specific spatial touring route possible. By comparing the time-of-stay measured and predicted, the ergonomic measurements of different design solutions, ENS makes effort to objectively and intuitively reflect the advantages and disadvantages of the space designs from the perspective of the user experience. ENS, like Space Syntax, attempts to connect research data with space design, as seen in the design of ski jumping centers for Beijing 2022, and the accuracy of ENS remains to be tested by time (Fig. 4. Ergonomic notation for Nordic Cluster, Beijing 2022_[Unfilled], Zhang Li et al. 2022).

Discussion

For the diagrams based on *topology*, there are three main targets these diagrams trying to achieve: (1) Comparability: comparing different design strategies using a topological prototype. (2) Calculability: inducing the perception quantitatively. (3) Evaluation-orientation: the ability of analyzing and evaluating a built space or a design solution. The highly concise topological structure enables the comparison and accurate calculation of various design projects. Meanwhile, the nodes and connections of topology imply the two typical ways of user experience, namely staying

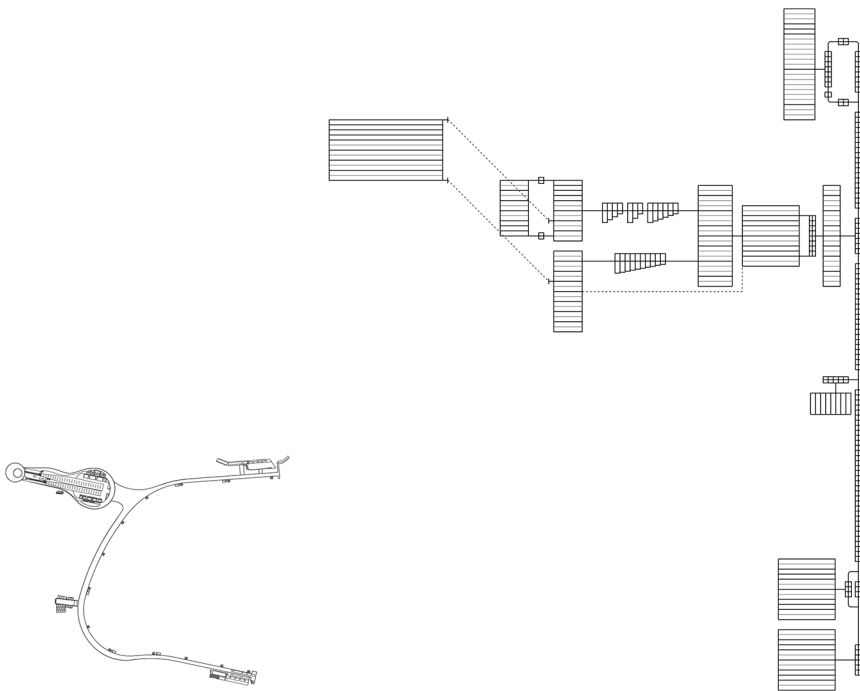


Fig. 4 Ergonomic notation for Nordic Cluster, Beijing 2022_[unfilled], which shows the predicted time-of-stay and the topological structure of the cluster. A figure from Zhang Li et al. (2022).

and passing. This type of diagrams simulates and predicts spatial experience quality through the trajectory and duration.

For the diagrams based on *encoding*, there are four distinguishing features: (1) Diachronism: simplifying complex movements into readable steps. (2) Dimension reduction: mapping multidimensional information onto a two-dimensional plan. (3) User-end: recording human performance from the subjective perspective rather than translating environmental elements. (4) Possibility of computer generation: using regular symbolic language to express irregular movements and different spatial experiences. The first two are about how dynamic human performance in space can be expressed on a static plane, bringing these diagrams to be recognized by the computer, as well as to participate in the form generation. Unlike objective map markers, these space-time data are presented in the order of people's perception, allowing them to be used to evaluate the space from the user-end.

If we juxtapose two types above, we can see Topology shows a strong ability and wide application in extracting spatial structure, while encoding is an efficient model to take time variable into consideration, both of which lay the foundation for translating space information and human performance into relevant parameters. While most of these diagrams are for research, devoted to recording, analyzing and clarifying the relationship between user behavior and space, we should note that the attempt to generate architecture design based on research data through diagrams has never stopped. From Generative Diagram to Space Syntax and then to ENS, it's mathematical models that enable the integration of analytical diagram and design diagram.

Conclusion

Diagrams based on certain mathematical models have the possibility to describe / simulate spatial experience from their ability in abstracting spatial form, measuring human perception in chronological order, and quantifying the relation in between.

Topology and Encoding are two of the most important models adopted in the description of spatial experience. *Diagrams based on Topology* performs well at simplifying overloading spatial information while *diagrams based on Encoding* are adept in expressing human perception sequentially.

The effort to combine data analyses and space designs using diagrams continues all the time. With the development of machine learning and wearable ergonomic technologies, accessible datasets used to train models and thus applied in diagrams for better spatial experience are way ahead and waiting for our exploration.

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Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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