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Design of the contingent royalty rate as related to the type of investment

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Abstract

This study investigates the design of the royalty rate in a first-price auction across three types of investments: incremental and lumpy with or without an exogenously given intensity. A bidder's investment cost comprises private information. This, together with the stochastic evolution of the price of the output generated from the auctioned project, precludes the seller from setting the exact dates of investment with the winner. However, the seller can set the royalty rate to equate the winner's royalty payment with the winner's information rent so that the winner acts as if to maximize the seller's revenue. We derive two main conclusions. First, compared with the case in which investment is lumpy with an exogenously given intensity, the seller can set a lower royalty rate on incremental investment because she can collect additional royalty payments from the winner, who has the option to later expand capacity. Second, the impact of output price uncertainty on the optimal royalty rate for the three types of investments exhibits two different patterns. When investment is either incremental or lumpy with an exogenously given intensity, greater output price uncertainty reduces the royalty rate. When investment is lumpy with variable intensity, greater output uncertainty raises the royalty rate. Our results imply that auctioneers may charge differential royalty rates for different types of investments.

Keywords: Cash payment, First-price auction, Incremental investment, Lumpy investment, Mechanism design, Real options, Royalty rate, Uncertainty

JEL Classification: D82, G11, G30, L20

Introduction

Most studies on security-bid auctions (e.g., Board 2007; Cong 2019, 2020) report that auctioneers can offer bidders the opportunity to invest in a lumpy investment project with an exogenously given intensity. As Bar-Ilan and Strange (1999) indicate, uncombinability and adjustment costs are two main reasons why investments exhibit lumpiness.[1] However, it is more appropriate to assume that investment is lumpy with variable intensity. For example, landowners must decide the time to change use and density of

[1] For example, land development is not combinable, because redeveloping a piece of land usually requires dismantling or substantially renovating an existing structure. Another example is that, when replacing equipment or renovating public infrastructure, fixed adjustment costs are likely to be generated.

development (Capozza and Li 1994; Jou and Lee 2007). Similarly, a power-generating company must decide when to replace its facility and the capacity level of a new plant. By contrast, other investments occur incrementally. For example, a government may offer bidders the opportunity to exploit minerals and later expand their investments (Trigeorgis 1993).

Auctioneers may offer cash bids at a given royalty rate. However, we contribute to the literature by developing a unified model to investigate the optimal royalty rate across the aforementioned three types of investments, including incremental and lumpy, with an exogenously given or variable intensity. Bar-Ilan and Strange (1999) find that the comparative statics and capital stock evolution of the first two types, which are quite similar, are strikingly different from the last one. This study investigates whether this pattern persists in contingent royalty rate design.

We construct a two-stage private value model (Vickrey 1961) in which a seller and multiple potential bidders are risk-neutral and maximize their expected payoffs.[2] In the first stage, the seller uses a direct revelation mechanism that includes the allocation rule, cash payments, and the royalty rate. Participating bidders have private information regarding their investment costs and bid cash through a first-price auction (FPA). After winning the bid, the winner has the option to engage in one of three types of investments. The equilibrium bid is derived using a perfect Bayesian Nash solution subject to the constraints of incentive compatibility and individual rationality. In the second stage, the winner faces a stochastic evolution in the price of the output generated by exercising the investment option. The winner chooses the timing and/or intensity of investment upon winning the auction and then delivers the royalty payment to the seller after producing the output. A postauction moral hazard problem arises as an unobservable type of bidder, and the stochastic evolution of the output price precludes the seller from contractually setting the exact dates of investment with the winner.

We solve the two-stage problem backwards. In the case of lumpy investment, in the second stage, the winner faces an investment decision, as described by McDonald and Siegel (1986) and Bar-Ilan and Strange (1999). The winner chooses the intensity of the investment on the date when the option value of waiting is equal to the sunk cost of investment. In the case of incremental investment, in the second stage, the winner faces a marginal investment decision, as described by Pindyck (1988).[3] The winner invests in the expansion option on the date on which the option value of investing in an additional unit of capital is equal to the marginal value of that unit, net of the unit cost of capital. Furthermore, in the first stage, the seller sets the royalty rate and participating bidders bid cash for the investment project. The winner maximizes his valuation of the investment less his royalty payment, whereas the seller maximizes her revenue, which equals the winner's valuation of the investment minus his information rent. Therefore, the seller should set the royalty rate to equate the winner's royalty payment with his information rent, with the winner acting as if to maximize the seller's revenue.

We derive the following two main conclusions: first, compared with the case in which investment is lumpy with an exogenously given intensity, the seller can set a lower

[2] For ease of exposition, the seller is female, and the bidders are male.

[3] See also Hagspiel et al. (2016) and Huisman and Kort (2015), both of which focus on a firm's capacity choice problem.

royalty rate on incremental investment because she can collect additional royalty payments from the winner, who has the option to later expand capacity. Second, the impact of output price uncertainty on the optimal royalty rate for the three types of investments exhibits two different patterns. When investment is either incremental or lumpy with an exogenously given intensity, greater output price uncertainty encourages the winner to wait for a more favorable output price to exercise the investment option, which raises the winner's royalty payment. Therefore, the seller can reduce the royalty rate. When the investment is lumpy with variable intensity, greater output uncertainty induces the winner to choose a larger investment intensity. Although this results in the seller receiving more royalty payments that are subject to decreasing marginal returns to capital, the benefit is outweighed by the increment of the information rent collected by the winner (embedded in the investment cost), which is linear in the capital stock. Consequently, greater uncertainty induces the seller to increase their royalty rate.

Our results have implications for auctions in terms of the opportunity to invest in long-lived investment projects, such as the Outer Continental Shelf (OCS) lease auctions in the United States. The United States Department of the Interior (DOI) uses auctions to allocate exploration and drilling rights for oil and gas to federal lands on the OCS (Hendricks and Porter 1988; Hendricks et al. 2003; Haile et al. 2010).[4] These lands are divided into tracts that have not been previously explored, or "wildcat tracts," and tracts adjacent to wildcat tracts in which a discovery has been made, or "drainage tracts." Since the introduction of the Area-Wide Leasing program in 1983, the federal offshore leasing program has sold leases using a first-price sealed bid auction with a reserve price. The winner of a lease pays the bid, or "bonus," on the sale date and 18.75% or 12.5% (16.67% prior to 2008) in royalties on revenues earned from post-sale production. If a wildcat tract lease is sold, the winner has the right to conduct exploratory drilling over a fixed period (typically 5 years). Most sales are wildcat sales (Hendricks and Porter 1996). The winner who leases a wildcat tract is likely to have the option to invest in an incremental project because he needs to decide the timing and initial intensity capacity. After making a discovery, he has the option of expanding capacity.[5] Thus, our study suggests that the DOI can charge a higher royalty rate for the wildcat tract when the uncertainty is smaller.

The rest of this study is organized as follows: "Literature review" section presents a literature review. "Model" section introduces the assumptions of this model. "Lumpy investment" section analyzes the winner's choice of lumpy investment with variable or an exogenously given intensity. For each case, we derive the optimal royalty rate and assess how various factors (including output price uncertainty) affect this rate. "Incremental investment" section is similar to "Lumpy investment" section for the case of incremental investment. "Managerial and policy implications" section describes how the auctioneer's design of the royalty rate relates to the type of investment and presents the policy and managerial implications of the theoretical results. Finally, "Conclusion" section offers concluding remarks and suggestions for future research.

[4] The lease auctions were conducted by the Minerals Management Service before 2010, then by the Bureau of Ocean Energy Management and the Bureau of Safety and Environmental Enforcement (Cong 2019).

[5] Hendricks et al. (1987) report that during 1954 and 1969, 71% of wildcat leases made a discovery.

Literature review

This study is closely related to the literature that employs the real options approach to investigate auction design, such as Board (2007) and Cong (2019, 2020).[6] Board (2007) considers a seller who auctions a dynamic option among many bidders. After the auction, the winning bidder, with private information regarding the value of the option, decides the date of executing the option when faced with the stochastic evolution of the investment cost over time. In this case, a revenue-maximizing auction consists of an upfront bid plus a contingent fee, whereby the fee is chosen in a Pigouvian manner. Thus, the winning bidder's choice of exercise time maximizes the seller's expected revenue. Furthermore, the type of the winning bidder determines the contingent fee. In contrast, this study demonstrates that the optimal contingent royalty rate is related to the parameters that characterize the stochastic evolution of the output price.

Cong (2019, 2020) assumed that bidders have private information about their investment costs. The winning bidder has the choice to execute an investment project with an exogenously given capacity that generates revenue evolving as a geometric Brownian motion. Cong (2020) reports that informal auctions take place before formal auctions. By contrast, Cong (2019) considers various selling mechanisms and shows that the optimal mechanism combines an upfront charge with a contingent royalty rate. He empirically demonstrates that a higher royalty rate delays the investment project's exercise. He provides an analytically tractable solution for the optimal royalty rate that is similar to ours, which is only applied to the case of lumpy investment with an exogenously given capacity.

This study is related to the literature on security-bid auctions (e.g., Krishna 2010), pioneered by DeMarzo et al. (2005). They analyze auctions in the context of asymmetric information, in which the winner's payment includes a share of the cash flow or (ex-post) value generated from the auctioned asset. They reveal that the seller's expected revenue is higher if the payment to the seller as a function of the realized value is steeper. In contrast, this study analyzes the choice of royalty rate in the presence of postauction moral hazard.[7]

Several articles on security-bid auctions either make assumptions or investigate topics that differ from ours in this study. These include: Liu (2016), Sogo (2017), Bernhardt et al. (2020), Liu and Bernhardt (2021), and Fioriti and Hernandez-Chanto (2021). Although we assume that bidders are ex ante identical, Liu (2016) analyzes the effects of heterogeneity in terms of bidders' valuation distribution and standalone values in equity

[6] See also Bhattacharya et al. (2022), and Herrnstadt et al. (2019). Bhattacharya et al. (2022) propose a common-value model of contingent payment auctions that explicitly links auction design to postauction economic activity in the context of Permian Basin oil auctions in New Mexico State. They reveal that the New Mexico State government can choose a royalty rate equal to 29% to maximize its revenue among all bonus auction formats. Herrnstadt et al. (2019) explicitly model the effects of the length of the primary term on drilling decisions and on the value received by the mineral owner and firm on the Haynesville Shale in northwestern Louisiana and East Texas. The primary term specified in the lease contract is the period of time the firm has to drill at least one well on the leased parcel. They determine that, for the section with drilling, 72%, 15%, 5%, and 4% of firms had one, two, three, and four or more wells drilled. This result suggests that the winning firm in the oil lease auction is provided with an opportunity to invest in the type of incremental investment project adopted in our framework.

[7] Our paper is thus also related to Samuelson (1987), Kogan and Morgan (2010), Laffont and Tirole (1987), Esö and Szentes (2007), Riordan and Sappington (1987), and McAfee and McMillan (1987). The first article indicates that adverse selection and moral hazard complicate the effect described in DeMarzo et al. (2005). The second presents a comparison of equity and debt auctions under moral hazard in an experimental study. The third derives an optimal linear incentive contract under competition, information asymmetry, and moral hazard. The remaining three articles focus on how post-auction decisions affect auction design.

auctions. Sogo (2017) considers a moral hazard problem inherent in the equity auctions of assets, in which, before an auction, the seller has private information about the possible returns on the auctioned assets. By contrast, we assume that bidders have private information about their investment costs.[8] Bernhardt et al. (2020) analyze optimal auction mechanisms when bidders, who base heterogeneous costly entry decisions on their valuations, pay with a fixed royalty rate and cash. However, we assume that bidders have the same entry costs. Fioriti and Hernandez-Chanto (2021) introduce risk-averse bidders to assess how security design affects bidders' equilibrium behavior as well as the revenue and efficiency of the auction. We assume that the seller and bidders are all risk-neutral. Finally, Liu and Bernhardt (2021) consider a setting in which potential merger partners are privately informed of their standalone values and merger synergies. We investigate formal rather than informal auctions.

Model

Assume that a risk-neutral seller and N risk-neutral potential bidders have the same discount rate ρ. When the seller auctions an option to invest in a project to these bidders, each bidder i knows his private investment cost of installing one unit of capital, θ_i, where the investment costs are completely irreversible as typically assumed in the real options literature (e.g., Dixit and Pindyck 1994).[9] Bidder i learns that the distribution of types for other bidders is independently and identically distributed with positive support $\left[\underline{\theta}, \overline{\theta}\right]$. The cumulative distribution and density function of θ_j are denoted by $F(\theta_j)$ and $f(\theta_j)$, respectively, where $\underline{\theta} \leq \theta_j \leq \overline{\theta},\ j = 1, \ldots, N$. Let $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_N)$ and $\boldsymbol{\theta}_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_N)$. An investment project that is never developed is worthless to the winner and seller.[10]

We compare an auctioneer's design of contingent royalty rates among three types of investments in a two-stage framework. In the first stage, the seller uses a direct revelation mechanism (Q_i, C_i, ϕ_i) that consists of an allocation function, upfront payments, and a royalty rate, respectively. The seller holds an auction through an FPA and sets the royalty rate at time $t = 0$. Each bidder i who announces his type $\tilde{\theta}_i$ and makes an upfront cash payment $C_i(\tilde{\boldsymbol{\theta}})$ is awarded the option to invest in a project with probability $Q_i(\tilde{\boldsymbol{\theta}})$, where $\tilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_N)$ and $\sum_{i=1}^{N} Q_i(\tilde{\boldsymbol{\theta}}) \leq 1$. In the second stage, if bidder i wins the auction, he must decide the timing of investment. If his investment is lumpy, he invests in either an exogenously given or variable intensity at $t \geq 0$. By contrast, if investment is incremental, he chooses a capacity at $t \geq 0$ and has the option to later increase the capacity. In either scenario, he is required to pay royalties after producing the output.

[8] Consequently, in our paper there is no need for the seller to induce an optimal ex-post game for the winning bidder as in Sogo (2017).

[9] As Lambrecht (2017) indicates, Dixit and Pindyck (1994) provide an overview of real options research up to the middle of the 1990s. From the middle of the 1990s onward, the focus of real options research shifted from developing to applying real options methods. The real options approach remains highly relevant today, as illustrated by a special issue of *Journal of Banking and Finance* (Lambrecht 2017) and a review article published in *Journal of International Business Studies* analyzing this approach (Chi et al. 2019). One may apply the real options approach to the recent types of investment such as Fintech investments (Kou et al. 2021) and solar energy-based transportation investment projects (Kou et al. 2022).

[10] Extending the model to the case in which the investment project is valuable despite never being developed is straightforward, as Board (2007) and Cong (2019) demonstrate.

We solve the two-stage problem backwards. We first solve the winner's optimal investment strategy. Next, we derive the bidding equilibrium and the seller's optimal choice of royalty rate. We consider both cases in which the investment is lumpy and incremental.

Lumpy investment

Suppose that the investment is lumpy. In the second stage, bidder i faces an investment problem, as described by McDonald and Siegel (1986) and Bar-Ilan and Strange (1999). Let $K_i(t)$ denote bidder i's stock of capital, and $Y_i(t)$ denote his level of output at time t after he has won the bid on date 0. Bidder i transforms each unit of capital, $K_i(t)$, to $K_i(t)^{\alpha}$ units of output per period using the production technology as follows:

$$Y_i(t) = K_i(t)^{\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where α is the productivity of capital, that is, the percentage change in output, given a 1% change in capital stock. The price of the output $P(t)$ evolves according to geometric Brownian motion, which is written as follows:

$$dP(t) = \mu P(t)dt + \sigma P(t)d\Omega(t), \tag{2}$$

where μ is the expected growth rate of $P(t)$, σ (>0) is the instantaneous volatility of this growth rate, and $d\Omega(t)$ is the increment of the standard Wiener process.[11]

Under the assumption that capital never depreciates and the marginal cost of production is constant, as denoted by w, bidder i's profit before royalty payment at time t is obtained as follows[12]:

$$\pi_i(K_i(t), P(t)) = K_i(t)^{\alpha}(P(t) - w). \tag{3}$$

Using Eq. (1): In the absence of any royalty payment, bidder i's value of investment at time t, $V_1(K_i(t), P(t))$, is equal to the expected present value of the return on capital from time t to infinity, beginning from the output price $P(t)$, that is,

$$V_1(K_i(t), P(t)) = \mathrm{E}\int_t^{\infty} e^{-\rho(s-t)}K_i(t)^{\alpha}[P(s) - w]ds = K_i(t)^{\alpha}\left[\frac{P(t)}{(\rho-\mu)} - \frac{w}{\rho}\right]. \tag{4}$$

We assume that the investment cost $\theta_i K_i(t)$ is not observable. This, together with the stochastic evolution of the output price, precludes the seller setting a contract with the winning bidder regarding the investment timing. Our assumption is realistic because, in practice, contracts or security designs based on profits are rare; profit-sharing is instead often combined with employee ownership and may be associated with other forms of financial participation and with varying levels of employee information and participation in decision-making (Estrin et al. 1997). Furthermore, following DeMarzo et al. (2005) and Bernhardt et al. (2020), we assume that the winning bidder incurs an upfront cost $X \geq 0$. This cost can be interpreted as initial resources required for the project, such as illiquid human capital, the cost of underwriting (in the case of security issuance), or

[11] One may assume that the output price follows a Possion jump process to capture the possibility of the catastrophic event such as the stock price crash risk as in Wen et al. (2019).

[12] If we assume that capital depreciates at a constant rate, then the impact of the depreciation rate on bidder i's value and choice of capital, the seller's revenue, and royalty rate is qualitatively the same as that of the discount rate.

simply the winning bidder's opportunity cost. In any case, X is common knowledge and invariable across bidders.[13]

The seller holds an auction on date 0, with $P_0 = P(0)$. The winning bidder, bidder i, invests on date τ_i and installs either an exogenously given stock of capital, $\overline{K}$, or a variable stock of capital equal to $K_i(\tau_i)$. After the date τ_i, at each instant t, bidder i delivers contingent payment $\phi_i(\tilde{\boldsymbol{\theta}})\overline{K}^{\alpha}P(t)$ or $\phi_i(\tilde{\boldsymbol{\theta}})K_i(\tau_i)^{\alpha}P(t)$ to the seller.[14]

Lumpy investment with variable intensity

Choices of investment timing and intensity

Consider the case in which the winning bidder installs a stock of capital, $K_i(\tau_i)$. We first derive the choice of investment timing and intensity in this case and then investigate how various exogenous parameters affect these two decisions. We investigate the design of the contingent royalty and bidding equilibrium in an FPA. Finally, we investigate the effect of various parameters on the optimal royalty rate.

Given that the output price is $P(t)$ and the royalty rate is ϕ_i, the value of investment at time t after bidder i executes the project, denoted by $V(K_i(\tau_i), P(t), \phi_i)$, is equal to the expected present value of the return on capital net of the royalty payment from time t to infinity, that is,

$$V(K_i(\tau_i), P(t), \phi_i) = \mathrm{E}\int_t^{\infty} e^{-\rho(s-t)}K_i(\tau_i)^{\alpha}[(1-\phi_i)P(s)-w]ds = K_i(\tau_i)^{\alpha}\left[\frac{(1-\phi_i)P(t)}{(\rho-\mu)} - \frac{w}{\rho}\right]. \tag{5}$$

The winner of type θ_i pays cash C_i at the time of the auction, $t = 0$, and pays royalties $\phi_i K_i(\tau_i)^{\alpha}P(t)$ at each instant $t \geq \tau_i$. Let us define $W(K_i(\tau_i), P(\tau_i), \phi_i, \theta_i) = V(K_i(\tau_i), P(\tau_i), \phi_i) - \theta_i K_i(\tau_i)$. The bidder i's valuation on the project net of royalty payments, investment costs, and cash payments (C_i) at $t = 0$ is then obtained as follows:

$$S_i(C_i, \phi_i, \theta_i) = \mathrm{E}e^{-\rho\tau_i}[W(K_i(\tau_i), P(\tau_i), \phi_i, \theta_i)] - X - C_i. \tag{6}$$

When the winner chooses the timing and intensity of the investment, the first term in Eq. (6), $\mathrm{E}e^{-\rho\tau_i}[W(K_i(\tau_i), P(\tau_i), \phi_i, \theta_i)]$, can be written as the expected discounted present value of the cash flows to bidder i over the infinite future beginning from τ_i as follows:

$$\begin{aligned}&\mathrm{E}e^{-\rho\tau_i}[W(K_i(\tau_i), P(\tau_i), \phi_i, \theta_i)]\\ &= \underset{K_i(\tau_i)}{\mathrm{Max}}\, \mathrm{E}\left\{\int_{\tau_i}^{\infty} e^{-\rho s}K_i(\tau_i)^{\alpha}[(1-\phi_i)P(s)-w]ds - \theta_i K_i(\tau_i)\right\}.\end{aligned} \tag{7}$$

Equation (7) indicates that when making the investment decision at time τ_i, bidder i also chooses the stock of capital, $K_i(\tau_i)$, which maximizes the net value of investment. As reported in the real options literature (Dixit and Pindyck 1994), we cannot determine the optimal value of τ_i, denoted by τ_i^*. Instead, we can derive the critical level of $P(\tau_i)$

[13] Che and Kim (2010) demonstrate that all the results related to security auctions are very sensitive to the assumptions regarding X. If X can vary from bidder to bidder, the relative performance of different securities depends on the nature of X. We abstract away from investigating this issue, since this issue is beyound the context of our model. We thank a referee for pointing out this.

[14] In the second-stage problem, we write $\phi_i(\tilde{\theta})$ as ϕ_i because the contingent royalty rate has been determined in the first stage and is thus exogenous to the winning bidder in this stage.

that triggers investment, denoted by P_i^*. When $P(\tau_i)$ reaches this trigger level, bidder i installs the stock of capital $K_i(\tau_i)$, as denoted by K_i^*. Proposition 1 is as follows:

Proposition 1 (Choices of timing and intensity of investment) *If we assume that the output price on the date of auction is sufficiently unfavorable, such that bidder i does not invest immediately, then his respective choices of investment timing and intensity are as follows*:

$$P_i^* = \frac{\beta_1(\rho-\mu)(1-\alpha)w}{(\beta_1(1-\alpha)-1)\rho(1-\phi_i)}, \tag{8}$$

and

$$K_i^* = \left[\frac{\alpha w}{(\beta_1(1-\alpha)-1)\rho\theta_i}\right]^{\frac{1}{(1-\alpha)}}, \tag{9}$$

where $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$ *and* $\beta_1(1-\alpha)$ *must be larger than one to ensure that* P_i^* *and* K_i^* *are positive.*[15] *The comparative static results are as follows*:

(*i*) *Bidder i delays investment* (P_i^* *increases*) *and installs a larger stock of capital* (K_i^* *increases*) *if either the output price uncertainty is greater* (σ *increases*) *or the productivity of capital is higher* (α *increases*).[16]
(*ii*) *If bidder i faces a higher royalty rate* (ϕ_i *increases*), *he does not change the stock of capital but delays investment.*
(*iii*) *Bidder i delays investment and installs a larger stock of capital if he incurs a higher marginal cost of production* (w *increases*).
(*iv*) *If bidder i expects the output price to increase more rapidly* (μ *is higher*) *or discounts the future less* (ρ *is lower*), *he installs a larger stock of capital but may accelerate, delay, or retain the investment date.*[17]
(*v*) *If bidder i incurs a higher investment cost* (θ_i *increases*), *he does not change the date of investment but installs a smaller stock of capital.*

Proof See "Appendix 1".

Determinants of the optimal royalty rate

Consider the problem in the initial stage, when the seller determines the optimal royalty rate and bidders employ their bidding strategies. If all other bidders report truthfully, bidder i chooses his report $\tilde{\theta}_i$ and the intensity of the investment at exercise time τ_i to maximize his *interim utility*, which is obtained as follows:

[15] Given that σ is negatively related to β_1, σ must vary from zero to a ceiling level.

[16] We assume that $K_i^* \geq 1$ for the comparative static result of α.

[17] An increase in μ or a decrease in ρ raises both the value of investing and the option value of waiting. An increase in μ encourages investment and a decrease in ρ discourages investment, thus leading to an indefinite effect on the investment timing.

$$
\begin{aligned}
&U_i(\theta_i, \tilde{\theta}_i, \tau_i) \\
&= \mathrm{E}_{\boldsymbol{\theta}_{-i}} \Bigg\{ Q_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i}) \underset{K_i(\tau_i)}{\mathrm{Max}}\, \mathrm{E}\Bigg[\int_{\tau_i}^{\infty} e^{-\rho s} K_i(\tau_i)^{\alpha} \Big((1-\varphi_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i}))P(s) - w\Big) ds \\
&\quad -\theta_i K_i(\tau_i)\Bigg] - X - C_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i}) \Bigg\}.
\end{aligned} \tag{10}
$$

The equilibrium utility of bidder i, under the assumption that the other bidders report honestly, is obtained through $T_i(\theta_i) = U_i(\theta_i, \theta_i, \tau_i^*(\theta_i, \phi_i(\theta_i, \boldsymbol{\theta}_{-i})))$. Incentive compatibility requires that $T_i(\theta_i) \geq U_i(\theta_i, \tilde{\theta}_i, \tau_i)$, that is, bidder i's utility is not reduced when the bidder reports honestly. Individual rationality requires $T_i(\theta_i) \geq 0$, that is, bidder i is not negatively affected by participating in the auction. Proposition 2 is as follows:

Proposition 2 *Let us define the value of* $W(K_i(\tau_i), P(\tau_i), \phi_i, \theta_i)$ *at date* $\tau_i = \tau_i^*$ *as just equal to X, that is,* $W(K_i(\tau_i^*), P(\tau_i^*), \phi_i^*, \hat{\theta}_i) = X$. *Suppose that* $E_{\theta_{-i}}\left[Q_i(\tilde{\theta}_i, \theta_{-i}) S_1(\tilde{\theta}_i)\right]$ *is decreasing in* $\tilde{\theta}_i$, *where* $S_1(\tilde{\theta}_i) = \mathrm{E}\left[\underset{K_i(\tau_i^*)}{\mathrm{Max}}\, \mathrm{E}e^{-\rho\tau_i^*} K_i(\tau_i^*)\right]$ *and* $\tau_i^* = \tau_i^*\left(\tilde{\theta}_i, \phi_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i})\right)$. *Thus, an optimal auction design is present in an FPA and is expressed as follows*:

$$
\phi_i^* = \left(1 + \frac{(1-\alpha)\theta_i f(\theta_i)\beta_1}{\alpha F(\theta_i)}\right)^{-1}, \tag{11}
$$

and

$$
C_i^*(\theta_i) = \frac{P_0^{\beta}}{\beta P^*(z(\theta_i))^{\beta-1}} - \int_{\theta_i}^{\bar{\bar{\theta}}} \left(\frac{1-F(\theta')}{1-F(\theta_i)}\right)^{N-1} \left[\frac{P_0}{P^*(z(\theta'))}\right]^{\beta} d\theta' - X, \tag{12}
$$

where $\bar{\bar{\theta}} = \min(\bar{\theta}, \hat{\theta})$, $z(\theta_i) = \theta_i + \big(F(\theta_i)/f(\theta_i)\big)$ *and* $P^*(z(\theta_i)) = \frac{\beta_1(\rho-\mu)}{(\beta_1-1)\rho}\big(w + \rho z(\theta_i) K_i^*(z(\theta_i))\big)$.[18]

Proof See "Appendix 2".

Although we derive the bidding equilibrium in a perfect Bayesian Nash concept in Proposition 2, we focus on investigating the design of the royalty rate (ϕ_i) in the following discussion. As indicated in "Appendix 2", the seller sets an optimal royalty rate, such that the winning bidder acts as if to maximize the seller's expected revenue. This requires that the expected present value of the royalties paid be equal to the information rent received by the winning bidder. Bidder i chooses the investment timing τ_i (≥ 0) and intensity K_i to maximize his net private valuation of the opportunity to invest in a project minus the expected present value of the royalty payment, as expressed in Eq. (7), where the expected present value of the royalty payment at time τ_i, calculated using Eq. (5), is expressed as follows:

$$
D_1(\phi_i, K_i(\tau_i), P(\tau_i)) = \frac{\phi_i P(\tau_i) K_i(\tau_i)^{\alpha}}{(\rho-\mu)}. \tag{13}
$$

[18] The incentive compatible condition requires that the virtual valuation of bidder i, namely $\theta_i + \big(F(\theta_i)/f(\theta_i)\big)$, is increasing with θ_i. A sufficient condition for this to hold is that the inverse hazard function $F(\theta_i)/f(\theta_i)$ is nondecreasing in θ_i, which we assume in this study.

In contrast, the seller's revenue equals bidder i's net private valuation of the opportunity to invest in a project minus his information rent, where the information rent at time τ_i, calculated using Eq. (35) in "Appendix 2", is expressed as follows:

$$D_2(\theta_i, K_i(\tau_i)) = \frac{F(\theta_i)}{f(\theta_i)} K_i(\tau_i). \tag{14}$$

The seller's strategy is to set $D_1(\phi_i, K_i(\tau_i), P(\tau_i)) = D_2(\theta_i, K_i(\tau_i))$, that is, to determine an optimal royalty rate that equates the winner's royalty payment with his information rent. Substituting $K_i(\tau_i) = K_i^*$ and $P(\tau_i) = P_i^*$, that is, the date on which bidder i exercises the option to invest in this equality, yields the optimal royalty rate, denoted by ϕ_i^*, as obtained using Eq. (11).

Differentiating ϕ_i^* with respect to α, μ, ρ, σ, and θ_i the results in Proposition 3 is obtained.

Proposition 3 (Optimal choice of royalty rate on lumpy investment with variable intensity) *The seller should charge a higher royalty rate* (ϕ_i^* *is higher*) *when capital productivity is higher* (α *is higher*), *the output price is expected to grow more rapidly* (μ *is higher*), *both the winner and seller discount the future less* (ρ *is lower*), *or the output price uncertainty is greater* (σ *is higher*).[19]

Bidder i chooses a larger stock of capital if (i) either the winner's productivity of capital or the uncertainty of the output price is greater, as suggested in Proposition 1(i); and (ii) the expected growth rate of the output price increases or the discount rate decreases, as suggested in Proposition 1(iv). This raises both bidder i's royalty payments and the information rent. However, the increment of bidder i's royalty payment, which is subject to decreasing marginal return on capital, is outweighed by his information rent, which is proportional to the stock of capital, because he has private information about his investment cost. Accordingly, the seller should increase the royalty rate, as Proposition 3 indicates.

Lumpy investment with an exogenously given intensity

Consider the case in which the winner undertakes a lumpy investment with an exogenously given intensity $\overline{K}$. Substituting $K_i(t) = \overline{K}$ into Eq. (25) in "Appendix 1" generates the critical level of the output price that triggers bidder i to invest, calculated as follows:

$$P_i' = \frac{\beta_1(\rho - \mu)}{(\beta_1 - 1)\rho(1 - \phi_i)} \left(w + \rho\theta_i \overline{K}^{(1-\alpha)} \right). \tag{15}$$

Imposing $P(\tau_i) = P_i'$ and $K_i(\tau_i) = \overline{K}$ into Eqs. (13), (14), and (15) yields the optimal royalty rate, denoted by ϕ_i', calculated as follows:

[19] In addition, the seller may increase, decrease, or retain the current royalty rate when the winner incurs a larger cost in investment (θ_i is higher). This is because an increase in the winner's cost of investment will lead to two effects that are offset with each other: (1) bidder i will install a smaller stock of capital, as suggested by Proposition 1(v), which decreases the optimal royalty rate; and (2) bidder i's information rent will not decrease, given that the inverse hazard function is non-decreasing in θ_i, which raises or does not affect the optimal royalty rate. Consequently, the optimal royalty rate may increase, decrease, or not be affected.

$$\phi_i' = \left(1 + \frac{f(\theta_i)\beta_1}{F(\theta_i)(\beta_1 - 1)}\left(\frac{w}{\rho \overline{K}^{(1-\alpha)}} + \theta_i\right)\right)^{-1}. \tag{16}$$

According to Proposition 4, we differentiate ϕ_i' from $\alpha, \mu, \rho, \sigma, \overline{K}, w$, and θ_i.

Proposition 4 (Optimal choice of royalty rate on a lumpy investment with an exogenously given intensity) *The seller should set a higher royalty rate when the output price is expected to grow less rapidly* (μ *is lower*), *both the winner and seller discount the future more* (ρ *is higher*), *output price uncertainty is reduced* (σ *is lower*), *the exogenously given stock of capital is larger* ($\overline{K}$ *is higher*), *or the marginal cost of production is lower* (w *is lower*).[20]

Proposition 4 applies mainly in this case because either a decrease in μ, σ, or w or an increase in ρ and $\overline{K}$ reduces the critical level of the output price that triggers bidder i to invest, which decreases bidder i's royalty payment. Consequently, the seller must increase her royalty rate.

Incremental investment

Capacity expansion decision

We repeat the procedures for lumpy investment with variable intensity, as described in "Lumpy investment with variable intensity" section. Let us retain all assumptions for the case of lumpy investment but consider the case in which investment is incremental. Bidder i installs capital stock immediately if the auction date $t = 0$ is sufficiently favorable. Otherwise, he waits until τ_{i0}, at which point $P(\tau_{i0})$ passes a threshold. After either date 0 or τ_{i0}, at each instant, t bidder i pays royalty $\phi_i(\tilde{\boldsymbol{\theta}})K_i(t)^\alpha P(t)$ to the seller.

As detailed in "Appendix 3", the critical level of the output price that triggers the winner to expand capacity is calculated as follows[21]:

$$P_i''(t) = \frac{\beta_1(\rho - \mu)}{(\beta_1 - 1)\rho(1 - \phi_i)}\left[\frac{\rho\theta_i}{\alpha K_i(t)^{\alpha - 1}} + w\right]. \tag{17}$$

The optimal trigger strategy in Eq. (17) yields an expression for $K_i''(t)$, which, using the terminology of Bertola and Caballero (1994), we refer to as the "desired" stock of capital, expressed as follows:

$$K_i''(t) = \left[\left(\frac{(\beta_1 - 1)(1 - \phi_i)P(t)}{\beta_1(\rho - \mu)} - \frac{w}{\rho}\right)\frac{\alpha}{\theta_i}\right]^{\frac{1}{(1-\alpha)}}. \tag{18}$$

The meanings of $P_i''(t)$ and $K_i''(t)$ are as follows. Suppose that the date of auction $t = 0$ is sufficiently favorable. At time 0, bidder i installs an initial capacity, $K_i''(0)$, which is obtained by substituting $P(t) = P(0) = P_0$ into Eq. (18).[22] If the output price

[20] In addition, the seller may increase, decrease, or retain the royalty rate when capital productivity is higher (α is higher) or when the winner incurs a larger investment cost (θ_i is higher).

[21] We assume $K_i(t) \geq 1$ so that $P_i''(t)$ increases with $K_i(t)$.

[22] In Eq. (17), we assume that $P(t)$ has a lower bound, $P_c = \frac{\beta_1(\rho-\mu)w}{(\beta_1-1)\rho(1-\phi_i)}$, and that $K_i''(t)$ is thus positive. If $P_0 < P_c$, then bidder i does not install the initial capacity until the time at which $P(t)$ first passes P_c.

$P(t)$ is higher than P_0, bidder i adds more capital, increasing $K_i(t)$ until $P_i''(t)$ in Eq. (17) increases to $P(t)$. In other words, when the output price reaches its historical high, bidder i adds capital.

Equation (17) indicates that bidder i delays installing new capital ($P_i''(t)$ increases) if the output price uncertainty is greater (σ increases), bidder i incurs higher investment costs (θ_i increases), or bidder i pays more royalty (ϕ_i increases). These comparative static results are qualitatively the same as those for lumpy investment with an exogenously given intensity.

Determinants of the optimal royalty rate

For lumpy investment, as illustrated in Eqs. (8) and (9), we must impose $w > 0$ to derive a positive value for the trigger price and optimal stock of capital. However, for the case of incremental investment, we consider $w = 0$, from which we can derive testable implications regarding the determinants of the optimal royalty rate. Proposition 5 is as follows:

Proposition 5 *Let us define the value of* $W_3(K_i(t), P(t), \phi_i, \theta)$ *as* X *at* $\theta = \hat{\theta}$, *that is,* $W_3(K_i, P_i'', \phi_i'', \hat{\theta}) = X$, *where* ϕ_i'' *is obtained using Eq.* (19). *Suppose that* $E_{\theta_{-i}}\left[Q_i(\tilde{\theta}_i, \theta_{-i})S_2(\tilde{\theta}_i)\right]$ *decreases in* $\tilde{\theta}_i$, *where* $S_2(\tilde{\theta}_i) = \left[\underset{\{K_i(\tau_i^*)\}}{\mathrm{Max}}\ \mathrm{E}\int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i^*} 1_{[dK_i(\tau_i^*)>0]} dK_i(\tau_i^*)\right]$ *and* $\tau_i^* = \tau_i^*\left(\tilde{\theta}_i, \phi_i(\tilde{\theta}_i, \theta_{-i})\right)$. *Thus, an optimal auction design is present in an FPA and is expressed as follows*:

$$\phi_i'' = \left[1 + \frac{(\theta_i\beta_1(\beta_1(1-\alpha)-1)+\alpha)f(\theta_i)}{\alpha(\beta_1(1-\alpha)-1)(\beta_1-1)F(\theta_i)}\right]^{-1} \tag{19}$$

and

$$\begin{aligned} C_i''(\theta_i) = &\int_{K_i}^{\infty}\left(\frac{P}{P''(z(\theta_i))}\right)^{\beta_1}\left[\alpha v_i^{\alpha-1}\left(\frac{P''(z(\theta_i))}{(\rho-\mu)}\right)-\theta_i\right]dv_i \\ &-\int_{\theta_i}^{\bar{\bar{\theta}}}\left[\frac{1-F(\theta')}{1-F(\theta_i)}\right]^{N-1}\underset{\{K_i(\tau_i^*)\}}{\mathrm{Max}}\ \mathrm{E}\left[\int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i^*} 1_{[dK_i(\tau_i^*)>0]} dK_i(\tau_i^*)\right]d\theta' - X, \end{aligned} \tag{20}$$

where $\bar{\bar{\theta}} = \min(\bar{\theta}, \hat{\theta})$, $\tau_i^* = \tau_i^*(\theta', \phi_i(\theta', \theta_{-i}))$, *and* $z(\theta_i) = \theta_i + \left(F(\theta_i)/f(\theta_i)\right)$.

Proof See "Appendix 4".

As detailed in "Appendix 4", bidder i chooses an exercise time τ_i (≥ 0) to maximize his net private valuation of the opportunity to invest in a project minus the expected present value of the royalty payment, where the expected present value of the royalty payment at time t (≥ 0), as calculated using Eq. (49), is expressed as follows[23]:

[23] We must multiply $W_3(K_i(t), P(t), \phi_i, \theta_i)$ in Eq. (49) by $\phi_i/(1-\phi_i)$ when deriving the option value resulting from the royalty payment at time t obtained through $\phi_i K_i(t)^{\alpha} P(t)$, because bidder i's revenue at time t is obtained through $(1-\phi_i)K_i(t)^{\alpha} P(t)$.

$$D_1(\phi_i, P(t)) = \frac{\phi_i P(t) K_i(t)^\alpha}{(\rho - \mu)} + \frac{\phi_i}{(1-\phi_i)} \int_{K_i(t)}^{\infty} b_2 \theta_i^{1-\beta_1} P(t)^{\beta_1} k_i^{(\alpha-1)\beta_1} dk_i, \tag{21}$$

where $b_2 = \left[\frac{\alpha(1-\phi_i)}{\beta_1(\rho-\mu)}\right]^{\beta_1} (\beta_1 - 1)^{(\beta_1 - 1)}$.

"Appendix 4" also indicates that the seller's revenue equals the winner's net private valuation of the opportunity to invest in a project minus his information rent, where the information rent at time t, as calculated using Eq. (54), is expressed as follows:

$$D_2(\theta_i, K_i(t)) = \frac{F(\theta_i)}{f(\theta_i)} K_i(t). \tag{22}$$

The seller should set $D_1(\phi_i, P(t)) = D_2(\theta_i, K_i(t))$ so that the winner acts as if to choose the exercise time that maximizes the seller's expected revenue. Substituting $P(t) = P_i''(t)$ into this equality yields the optimal royalty rate, denoted by ϕ_i'', obtained from Eq. (19). Proposition 6 is as follows:

Proposition 6 (Optimal choice of royalty rate on incremental investment) *The seller should charge a higher royalty rate* (ϕ_i'' *is higher*) *for incremental investment when the growth rate of the output price is expected to decrease* (μ *is lower*), *the discount rate increases* (ρ *is higher*), *or the output price uncertainty is reduced* (σ *is lower*).[24]

Proof See "Appendix 5".

Equations (58)–(60) indicate that a smaller appreciation of the output price, a larger discount rate, or reduced uncertainty in the output price reduces the critical level of the output price that triggers the winner to add capital, which decreases the winner's royalty payment but does not change his information rent. Thus, the seller should increase the royalty rate, as Proposition 6 indicates.

Managerial and policy implications

Auctioneer's design of the royalty rate across the investment types

Our results extend the research on security-bid auctions, such as that of Board (2007) and Cong (2019). Board (2007) assumes that a bidder's valuation of the option to invest is private information and demonstrates that the seller can set a contingent payment that only depends on the type of bidder. In addition, Cong (2019) adopts a model that resembles our case of lumpy investment, with an exogenously given intensity and zero variable cost. He derives an optimal royalty rate that resembles the ϕ_i^* in Eq. (11). However, he does not examine how the underlying parameters affect this rate.

[24] In addition, the relation between each of the levels of capital productivity (α) or the unit cost of capital (θ_i) and the optimal royalty rate is indeterminate. As stated, the optimal royalty rate equates the winner's royalty payment with his information rent. Eq. (56) indicates that an increase in the winner's productivity of capital (a higher α) may increase, decrease, or not change the critical level of the output price that triggers the winner to expand capacity, which ambiguously affects his royalty payment but does not change his information rent. Thus, the seller may increase, decrease, or retain the royalty rate. Eq. (57) indicates that although an increase in the winner's investment cost (a higher θ_i) raises the critical level of the output price that triggers the winner to expand capacity and thus increases his royalty payment, his information rent either increases or remains unchanged, because $\partial\left(F(\theta_i) / f(\theta_i)\right) / \partial\theta_i \geq 0$. Thus, the seller may increase, decrease, or retain the royalty rate.

Table 1 Summary of the comparison of the optimal royalty rate across three investment types and the effect on the optimal royalty rate of increasing one variable when all others remain fixed

Variable	ϕ_i^*	ϕ_i'	ϕ_i''
Capital productivity, α	+	?	?
Expected growth rate of output price, μ	+	−	−
Discount rate, ρ	−	+	+
Output price volatility, σ	+	−	−
Unit cost of capital, θ_i	?	?	?
Fixed scale of operation, $\overline{K}$	N.A.	+	N.A.
Marginal cost of production, w	0	−	N.A.

ϕ_i^*, ϕ_i', and ϕ_i'' are the optimal royalty rate for lumpy investment with variable intensity, lumpy investment with an exogenously given intensity, and incremental investment, respectively. If $w = 0$, then $\phi_i' > \phi_i''$.

+ indicates that an increase in the variable causes the optimal royalty rate to increase.

− indicates that an increase in the variable causes the optimal royalty rate to be reduced.

? indicates that the relationship is uncertain.

0 indicates no effect.

N.A. indicates that the result is not applicable.

Table 1 presents a comparison of the optimal royalty rate across the three types of investments and summarizes all the comparative static results stated in Propositions 3, 4, and 6. The comparative static results of the optimal royalty rate with respect to μ, ρ, and σ for the case of lumpy investment with an exogenously given intensity are similar to those for incremental investment but are opposite to those for lumpy investment with variable intensity. In other words, a bidder's flexibility in choosing the intensity of investment affects the auctioneer's design of the royalty rate. This parallels the findings of Bar-Ilan and Strange (1999), which indicate that the comparative static results of the timing of investment and the evolution of capital stock are strikingly different when comparing lumpy investment with and without choice of intensity.

We focus on output price uncertainty σ to explain the rationale. An increase in output price uncertainty reduces the optimal royalty rate when the winner does not have the option to determine the intensity of investment, as assumed in Cong (2019), but increases the optimal royalty rate when the winner can determine the intensity. The reason is that when the investment intensity is exogenously given, the winner who faces greater output price uncertainty delays investment until a more favorable output price is presented and thus pays more royalties and receives the same information rent. Consequently, the seller needs to reduce the royalty rate to ensure that the winner chooses a date of investment that maximizes her expected revenue. When the bidder has the flexibility to choose the intensity, an additional effect should be considered; an increase in output price uncertainty increases the optimal stock of capital, as stated in Proposition 1(i). As explained in "Determinants of the optimal royalty rate" section, this raises the winner's royalty payment to a less extent than his information rent, such that the seller must increase the royalty rate. This adds effect more than offsets the effect on the optimal royalty rate resulting from the impact of output price uncertainty on the optimal date of investment. We thus contribute to the literature by demonstrating that, when designing the contingent royalty rate, auctioneers must consider not only the uncertainty embedded in the auctioned project but also whether the winner can choose the intensity of investment as he wishes.

We may compare the optimal royalty rate for the case of lumpy investment with an exogenously given intensity ϕ_i' with that of incremental investment ϕ_i'' in the special case where the marginal cost, w, is equal to zero. Imposing the condition of $w = 0$ onto ϕ_i' in Eq. (16) and comparing the result with ϕ_i'' in Eq. (19) yields $\phi_i' > \phi_i''$.[25] This result indicates that the seller should set a higher royalty rate when investment is lumpy with an exogenously given intensity compared to when investment is incremental, because the seller expects to collect additional royalty payments from bidder i when he has the option to incrementally add capital, as determined by comparing Eqs. (21) and (13).

Policy implications

The aforementioned results have implications for auctions on the opportunity to invest in long-lived investment projects such as OCS lease auctions in the United States. As explained in "Introduction" section, the leased lands are divided into tracts that have not been previously explored (i.e., wildcat tracts) and tracts adjacent to wildcat tracts in which a discovery has been made (i.e., drainage tracts). The winner of a lease pays the bid (i.e., bonus) on the sale date and 18.75% or 12.5% (16.67% prior to 2008) in royalties on revenues earned from post-sale production. Most sales are wildcat sales (Hendricks and Porter 1996). The winner who leases the wildcat tract undertakes an incremental project. He decides the timing and initial capacity when he discovers any minerals worth exploiting. After he makes a valuable discovery, he has the option of incrementally increasing the capital.[26] Thus, the results of Propositions 4 and 6 suggest that the DOI may charge differential royalty rates for wildcat tracts that face different oil price uncertainties.[27]

We may investigate whether the DOI has changed the OCS royalty rates using oil price volatility as a proxy for the CBOE Crude Oil ETF Volatility Index (MacroMicro n.d. OVX, https://en.macromicro.me/collections/4536/volatility/21526/ovx). The DOI has changed OCS royalty rates three times since 2007.[28] First, in 2007 and 2008, the DOI made major adjustments to the OCS royalty rates. At that time, the shallow water (200 m or less) rate was 16.67%, whereas the deepwater rate was generally 12.5%. Initially, the deepwater rate increased to 16.67%. Soon after, the rates of new leases at all water depths increased to 18.75%. The CBOE Crude Oil ETF Volatility Index rose from a trough of 27

[25] We use Eq. (55) to derive the analytically tractable solution of ϕ_i'' in Eq. (19). The second term on the right-hand side of Eq. (55) is equal to the first term on the right-hand side of Eq. (49) multiplied by $\phi_i \big/ (1 - \phi_i)$, assuming that $w = 0$. If we do not assume $w = 0$, then we need to solve for ϕ_i, which is in the integral of the first term on the right-hand side of Eq. (49). As a result, we will not be able to drive the closed-form solution of ϕ_i^{**} in Eq. (19). Instead, we need to solve ϕ_i^{**} by numerical analysis. We would like to derive a sharp result so that we assume that $w = 0$.

[26] The distinction between a wildcat and drainage lease may arguably be the quality of information about potential well productivity, which means that after exploratory drilling has been conducted, a wildcat lease is effectively a drainage lease, in terms of the type of investment per se. As Trigeorgis (1993) suggests, natural resource industries such as mine operations typically have the option to defer or to alter the operation scale. Both lessees of wildcat and drainage tracts have the option to defer. However, within the same term of a lease, lessees of drainage tracts are more likely to have the option to alter the operation scale than lessees of wildcat tracts.

[27] Royalties should be different for exploratory and drainage tracts, because exploration generates positive externalities both for neighboring tracts and for future investment on the same tract. These externalities are not accounted for fully by the firm because of revenue sharing. Generally, incremental investment decisions affect the continuation value of the object sold, but, as with the oil example, the firm does not internalize this effect. We thank one referee for providing this insight.

[28] Humphries (2017) reported the first two changes.

in 2007 to a peak of 96 in 2008. Consequently, royalties tend to be positively related to uncertainty in oil prices. Second, the DOI reduced the OCS royalty rate from 18.75% to 12.5% for shallow water blocks in lease sales held in the Gulf of Mexico on August 16, 2017. The CBOE Crude Oil ETF Volatility Index was approximately 26. Finally, on April 15, 2022, the DOI increased the royalty rate from 12.5% to 18.75% for new shallow water blocks in lease sales (The Associated Press April 16, 2022, https://www.npr.org/2022/04/16/1093195479/biden-federal-oil-leases-royalties). The CBOE Crude Oil ETF Volatility Index was approximately 52. Consequently, none of the changes fit our prediction in Proposition 6 because the DOI had considerations different from ours.[29]

Managerial implications

Our analysis has two implications for managers who wish to attend contingent royalty auctions. First, the optimal bidding rule depends on the investment type. Failing to conform to the bidding rule results in a manager bidding too low and therefore losing the bid or bidding too high and receiving a suboptimal payoff. Second, after winning the auction, the winner's investment rule depends on the investment type. A thorough understanding of the investment rule can provide managers with the opportunity to obtain the maximum investment payoff.

Conclusion

In this study, we compare the design of contingent royalty rates among the following three types of investment: (1) lumpy investment with variable intensity, (2) lumpy investment with an exogenously given intensity, and (3) incremental investment. The first involves a two-dimensional model, because the winner chooses investment timing and intensity. The other two involve a one-dimensional model in which the winner chooses either the timing or intensity of the investment, but not both. This study has two main findings. First, the seller should set a lower royalty rate for the winner and provide them with the opportunity to add capital incrementally. Second, output price uncertainty increases the optimal contingent royalty rate in the case of the two-dimensional model but decreases it in the case of the two one-dimensional models. The aforementioned results suggest that auctioneers must be cautious regarding auctioned investment types that may exhibit characteristics of capital irreversibility and uncertainty, as emphasized in the real options literature.

Our findings are based on the following simplified assumptions. First, the seller holds the auction on a prespecified date. Second, each bidder has private information regarding his investment costs and submits an independent bid. Third, all the bidders have the same entry costs. Fourth, all bidders are risk neutral. Finally, the auctioned investment project can be completed instantly, and the investment costs incurred once are completely irreversible.

Future studies could relax these assumptions to determine whether our findings are valid. First, bidders can choose the date on which an auction is held (Cong 2020) or have

[29] The DOI's explanation for its decision to drop the royalty rate on 16 August 2017 centered on the declining leasing, drilling, and production in shallow water. The DOI also indicated that oil and gas resource estimates in shallow water had declined in recent years, and low oil and natural gas prices and the marginal nature of the remaining Gulf of Mexico shelf resources supported the need for a lower rate.

common expectations regarding the value of the auctioned project (Bhattacharya et al. 2022). Second, bidders can be provided with time to construct an investment project (Aguerrevere 2003) or be allowed to partially recover the cost of their investment (Jou 2022; Kandel and Pearson 2002). Third, bidders can be assumed to have different entry costs, as in Bernhardt et al. (2020). Fourth, bidders could be assumed to be risk-averse, as in Fioriti and Hernandez-Chanto (2021). Finally, the role of joint bids can be considered, for instance, in US OCS auctions (Haile et al. 2010).

Appendix 1: Derivation of P_i^* and K_i^*

The value of investment before bidder i exercises the project is obtained through $F(K_i(t), P(t), \phi_i)$, where $t \le \tau_i$. Applying Itô's lemma, contingent claims analysis, and the required boundary conditions, we obtain the solution of $F(\cdot)$ and the threshold level of the output price that triggers investment $P'(t)$, as follows:

$$F(K_i(t), P(t), \phi_i) = b_0 P(t)^{\beta_1}, \tag{23}$$

where b_0 and $P'(t)$ are calculated as

$$b_0 = \frac{(1-\phi_i)K_i(t)^{\alpha}}{\beta_1(\rho-\mu)} P'(t)^{1-\beta_1}, \tag{24}$$

and

$$P'(t) = \frac{\beta_1(\rho-\mu)}{(\beta_1-1)\rho(1-\phi_i)}\left(w + \rho\theta_i K_i(t)^{1-\alpha}\right). \tag{25}$$

and β_1 is the larger root of

$$\phi(\beta) = -\frac{1}{2}\sigma^2\beta(\beta-1) - \mu\beta + \rho = 0. \tag{26}$$

$P'(t)$ in Eq.(25) applies to lumpy investment regardless of whether the intensity is exogenously given or not. If the intensity is variable, then the optimal $K_i(t)$, denoted by $K_i'(t)$, maximizes the option value to invest, $F(K_i(t), P(t), \phi_i)$, and therefore satisfies the first-order condition obtained through $dF(K_i(t), P(t), \phi_i)\big/ dK_i = 0$. Solving this condition yields the following equation:

$$K_i'(t) = \left[\frac{\alpha}{\theta_i}\left(\frac{(1-\phi_i)P(t)}{(\rho-\mu)} - \frac{w}{\rho}\right)\right]^{\frac{1}{(1-\alpha)}}. \tag{27}$$

Substituting $P(t) = P'(t)$ into Eq. (27) and $K_i(t) = K_i'(t)$ into Eq. (25) and solving these two equations simultaneously yields the solution to $K_i'(t)$, denoted by K_i^*, which is expressed as

$$K_i^* = \left[\frac{\alpha w}{(\beta_1(1-\alpha)-1)\rho\theta_i}\right]^{\frac{1}{(1-\alpha)}}. \tag{28}$$

This implies that K_i^* is finite when $\beta_1(1-\alpha) > 1$. Furthermore, the solution to $P'(t)$ is expressed as

$$P_i^* = \frac{\beta_1(\rho - \mu)(1-\alpha)w}{(\beta_1(1-\alpha)-1)\rho(1-\phi_i)}. \tag{29}$$

The comparative static results of Proposition 1 are obtained when we differentiate P_i^* and K_i^* with respect to $\alpha, \sigma, \phi_i, w, \mu, \rho$, and θ_i.

Q.E.D.

Appendix 2: Proof of Proposition 2

Applying the revelation principle as described by Board (2007) and Milgrom and Segal (2002), we transform the indirect mechanism that implements the contrainsed optimal mechanism into the direct mechanism. More precisely, we derive the optimal choice of $\tilde{\theta}_i$ as θ_i; hence, bidder i 's equilibrium utility is obtained as follows:

$$T_i(\theta_i) = \mathrm{E}_{\boldsymbol{\theta}_{-i}}\left\{\int_{\theta_i}^{\bar{\theta}} Q_i(\varepsilon, \boldsymbol{\theta}_{-i}) S_1(\varepsilon) d\varepsilon\right\} + T_i(\bar{\theta}), \tag{30}$$

where $S_1(\varepsilon) = \mathrm{E}\left[\underset{K_i(\tau_i^*)}{\mathrm{Max}}\, \mathrm{E}e^{-\rho\tau_i^*} K_i(\tau_i^*)\right]$, $\tau_i^* = \tau_i^*(\varepsilon, \phi_i(\varepsilon, \boldsymbol{\theta}_{-i}))$, and ε vary from θ_i to $\bar{\theta}$. In Eq. (30), $T_i(\tilde{\theta}_i)$ must be decreasing in $\tilde{\theta}_i$, which suggests that a bidder who reports a lower cost has a higher probability of winning the auction. Taking expectations over bidder i's type, θ_i, and integrating Eq. (30) by parts yields bidder i's ex-ante utility before the auction takes place, which is calculated as follows:

$$\mathrm{E}_{\theta_i}[T_i(\theta_i)] = \mathrm{E}_{\boldsymbol{\theta}}\left\{Q_i(\boldsymbol{\theta})\left[\underset{K_i(\tau_i^*)}{\mathrm{Max}}\, \mathrm{E}e^{-\rho\tau_i^*} K_i(\tau_i^*)\frac{F(\theta_i)}{f(\theta_i)}\right]\right\} + T_i(\bar{\theta}) \tag{31}$$

To verify the incentive compatibility condition, select $(\theta_i, \tilde{\theta}_i)$, where $\theta_i < \tilde{\theta}_i$.

$$\begin{aligned} U_i\left(\theta_i, \tilde{\theta}_i, \tau_i^*(\tilde{\theta}_i, \phi_i(\tilde{\theta}_i, \theta_{-i}))\right) &= T_i(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} \frac{\partial}{\partial\theta_i} U_i\left(\alpha, \tilde{\theta}_i, \tau_i^*(\alpha, \phi_i(\tilde{\theta}_i, \theta_{-i}))\right) d\alpha \\ &\leq T_i(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} \frac{\partial}{\partial\theta_i} U_i\left(\alpha, \alpha, \tau_i^*(\alpha, \phi_i(\alpha, \tilde{\theta}_i, \theta_{-i}))\right) d\alpha \\ &= T_i(\theta_i), \end{aligned} \tag{32}$$

where the inequality comes from the monotonic condition stated in Proposition 2, and the equality is derived from Eq. (30). To verify individual rationality, Eq. (30) implies that $T_i(\theta_i)$ is decreasing in θ_i. Hence, $T_i(\bar{\theta}_i) \geq 0$ implies that $T_i(\theta_i) \geq 0$.

Welfare at $t = 0$ is defined as the sum of the bidders' utilities and the seller's revenue at $t = 0$. Welfare, denoted by Y_1, thus equals the expected net present value of the option to invest net of the upfront cost, which is expressed as follows:

$$Y_1 = \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\boldsymbol{\theta}) \mathrm{E}e^{-\rho\tau_i^*}\left[\left(V_1(K_i(\tau_i^*), P(\tau_i^*)) - \theta_i K_i(\tau_i^*)\right) - X\right]\right\}, \tag{33}$$

where $V_1(K_i(t), P(t))$ is defined in Eq. (4). The seller's revenue in an auction with competitive bids at $t = 0$, denoted by R_1, is equal to the welfare obtained using Eq. (33)

minus the sum of all bidders' utilities, as obtained using Eq. (31) (see Board 2007), and is calculated as follows:

$$\begin{aligned} R_1 &= \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\boldsymbol{\theta})\mathrm{E}\left[e^{-\rho\tau_i^*}\left(V_1(K_i(\tau_i^*), P(\tau_i^*)) - \theta_i K_i(\tau_i^*)\right) - X\right] - \sum_{i=1}^{N} T_i(\theta_i)\right\} \\ &= \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\theta)\mathrm{E}\left[e^{-\rho\tau_i^*}(V_1(K_i(\tau_i^*), P(\tau_i^*)) - z(\theta_i)K_i(\tau_i^*))\right] - X\right\} - \sum_{i=1}^{N} T_i(\bar{\theta}), \end{aligned} \tag{34}$$

where $z(\theta_i) = \theta_i + \left(F(\theta_i)/f(\theta_i)\right)$ is the "virtual" valuation of bidder i. Incentive compatibility requires that $z(\theta_i)$ be increasing in θ_i. A sufficient condition for this relation to hold is that the inverse hazard function, $F(\theta_i)/f(\theta_i)$, is nondecreasing in θ_i, where $\left(F(\theta_i)/f(\theta_i)\right)K_i(\tau_i^*)$ is the information rent received by bidder i.

The seller chooses (Q_i, C_i, ϕ_i) to maximize the revenue obtained through Eq. (34) subject to the incentive compatibility condition (32) and the individual rationality where bidder i chooses $\tau_i^*(\theta_i, \phi_i(\theta_i, \theta_{-i}))$ to maximize $U_i(\theta_i, \tilde{\theta}_i, \tau_i)$, as obtained through Eq. (10). Imposing the conditions of $\tilde{\theta}_i = \theta_i$ and $\tau_i = \tau_i^*(\theta_i, \phi_i(\theta_i, \theta_{-i}))$ on $U_i(\theta_i, \tilde{\theta}_i, \tau_i)$ results in bidder i choosing the exercise date to maximize R_1 if the following condition holds:

$$\frac{\phi_i P(\tau_i) K_i(\tau_i)^{\alpha}}{(\rho - \mu)} = \frac{F(\theta_i)}{f(\theta_i)} K_i(\tau_i). \tag{35}$$

Note that the seller was not able to extract the winning bidder's surplus because the winning bidder has private information regarding the investment cost. However, the seller is able to align the winning bidder's interest with her own when Condition (35) holds. In equilibrium, the interim utility of bidder i, as expressed in Eq. (10), must be equal to his ex-post utility, as expressed in Eq. (30). The individual rationality condition requires that the ex-post utility of the least favorable type be equal to zero, $T_i(\bar{\theta}) = 0$. Using these two conditions and Eq. (35) and assuming that the perfect Bayesian Nash equilibrium holds, we determine that bidder i's cash payment in an FPA is obtained through $C_i^*(\theta_i)$ in Eq. (12).

Q.E.D.

Appendix 3: Derivation of $P_i''(t)$

We first derive bidder i's ex-post value, that is, his value after an auction takes place. Following Pindyck (1988) and Kandel and Pearson (2002), given bidder i's stock of capital $K_i(t)$ and the output price $P(t)$, his gross private valuation of the project is calculated as follows:

$$W_1(K_i(t), P(t)) = V_1(K_i(t), P(t)) + F_1(K_i(t), P(t)), \tag{36}$$

where on the right-hand side of Eq. (36), the first term is the value of the project, assuming bidder i does not add any capital, and the second term is the value of the option to install more capital. Define the value of an additional unit of capital as $\Delta V_1(K_i(t), P(t)) = \partial V_1(K_i(t), P(t))/\partial K_i(t)$ and the value of the option to add this unit of capital as $\Delta F_1(K_i(t), P(t)) = -\partial F_1(K_i(t), P(t))/\partial K_i(t)$, given that bidder i has already installed $K_i(t)$ units of capital. Using these definitions, we can write

$$W_1(K_i(t), P(t)) = \int_0^{K_i(t)} \Delta V_1(k_i(t), P(t))dk_i(t) + \int_{K_i(t)}^{\infty} \Delta F_1(k_i(t), P(t))dk_i(t). \quad (37)$$

Let $\Delta W_1(K_i(t), P(t)) = \partial W_1(K_i(t), P(t)) \big/ \partial K_i(t) = \Delta V_1(K_i(t), P(t)) - \Delta F_1(K_i(t), P(t))$. The term $\Delta V_1(K_i(t), P(t))$ must equal the expected present value of the marginal return to capital from time t to infinity, beginning from the output price $P(t)$, that is,

$$\Delta V_1(K_i(t), P(t)) = E_t \int_t^{\infty} e^{-\rho(s-t)} \frac{\partial \pi_i(K_i(s), P(s))}{\partial K_i(s)} ds = \alpha K_i(t)^{\alpha-1} \left[\frac{P(t)}{(\rho - \mu)} - \frac{w}{\rho} \right]. \quad (38)$$

Let

$$\Delta W_2(K_i(t), P(t), \phi_i) = \Delta V_2(K_i(t), P(t), \phi_i) - \Delta F_2(K_i(t), P(t), \phi_i), \quad (39)$$

where

$$\Delta V_2(K_i(t), P(t), \phi_i) = \Delta V_1(K_i(t), P(t)) - \Delta V_1(K_i(t), \phi_i P(t)) \quad (40)$$

and

$$\Delta F_2(K_i(t), P(t), \phi_i) = \Delta F_1(K_i(t), P(t)) - \Delta F_1(K_i(t), \phi_i P(t)), \quad (41)$$

such that

$$\Delta W_2(K_i(t), P(t), \phi_i) = \Delta W_1(K_i(t), P(t)) - \Delta W_1(K_i(t), \phi_i P(t)). \quad (42)$$

The winner of type θ_i pays cash C_i at the time of auction, $t = 0$, and pays royalties $\phi_i K_i(t)^{\alpha} P(t)$ after he installs the initial capacity at τ_{i0}, where $t \geq \tau_{i0}$. Bidder i 's gross valuation net of the royalty payments, investment costs, and cash payments (C_i) on date 0 is then calculated as follows:

$$S_i(C_i, \phi_i, \theta_i) = \mathrm{E}e^{-\rho\tau_{i0}}[W_3(K_i(\tau_{i0}), P(\tau_{i0}), \phi_i, \theta_i)] - C_i - X, \quad (43)$$

where $W_3(K_i(t), P(t), \phi_i, \theta_i) = W_2(K_i(t), P(t), \phi_i) - \theta_i K_i(t)$ and its expected value at $t = 0, \mathrm{E}e^{-\rho\tau_{i0}}[W_3(K_i(\tau_{i0}), P(\tau_{i0}), \phi_i, \theta_i)]$ can be written as follows as the expected present value of the cash flows to bidder i over the infinite future beginning from τ_{i0}:

$$\begin{aligned} &\mathrm{E}e^{-\rho\tau_{i0}}[W_3(K_i(\tau_{i0}), P(\tau_{i0}), \phi_i, \theta_i)] \\ &= \underset{\{K_i(\tau_i)\}}{\mathrm{Max}}\ \mathrm{E}\left\{ \int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i}[(1-\phi_i)K_i(\tau_i)^{\alpha}(P(\tau_i) - w)d\tau_i - \theta_i(1_{[dK_i(\tau_i)>0]})dK_i(\tau_i)] \right\}, \end{aligned} \quad (44)$$

where τ_i is the moment when new capital is installed, and $1_{[\cdot]}$ is an indicator function that is equal to one if new capital is installed and zero otherwise. Equation (44) indicates that when making a long-run investment decision, bidder i must choose the cumulative capital stock process, $K_i(t)$, that maximizes his net value of investment.

Applying Itô's lemma, contingent claims analysis, and the necessary boundary conditions, we obtain the solution for $\Delta F_2(K_i(t), P(t), \phi_i)$ in Eq. (41) and the threshold level of $P(t)$ that triggers investment, $P_i''(t)$, through the following calculation:

$$\Delta F_2(K_i(t), P(t), \phi_i) = b_1 P(t)^{\beta_1}, \tag{45}$$

where β_1 is defined in Eq. (25),

$$b_1 = \left[\alpha K_i(t)^{\alpha-1}\left(\frac{(1-\phi_i)P_i''(t)}{(\rho-\mu)} - \frac{w}{\rho}\right) - \theta_i\right] P_i''(t)^{-\beta_1} \tag{46}$$

and

$$P_i''(t) = \frac{\beta_1(\rho-\mu)}{(\beta_1-1)\rho(1-\phi_i)}\left[\frac{\rho\theta_i}{\alpha K_i(t)^{\alpha-1}} + w\right]. \tag{47}$$

Q.E.D.

Appendix 4: Proof of Proposition 5

After solving for bidder i's desired stock of capital, $K_i''(t)$ in Eq. (18), we derive his expected net valuation of investment. The option value to bidder i of installing an additional unit of capital $\Delta F_2(\cdot)$ is equal to the probability of reaching $P_i''(t)$, given that the current state is $P(t)$, $(P(t)/P_i''(t))^{\beta_1}$, multiplied by the net value of installing this unit immediately. Substituting b_1 in Eq. (46) into Eq. (45) yields the following equation:

$$\Delta F_2(K_i(t), P(t), \phi_i) = \left(\frac{P(t)}{P''(t)}\right)^{\beta_1}\left[\alpha K_i(t)^{\alpha-1}\left(\frac{(1-\phi_i)P''(t)}{(\rho-\mu)} - \frac{w}{\rho}\right) - \theta_i\right]. \tag{48}$$

Substituting Eqs. (40) and (48) into (39) and integrating the first and second terms on the right-hand side yields $V_2(K_i(t), P(t), \phi_i)$ and $F_2(K_i(t), P(t), \phi_i)$, respectively. Summing these two terms and subtracting the term $\theta_i K_i(t)$ yields bidder i's net valuation of the investment project, which is expressed as

$$\begin{aligned} &W_3(K_i(t), P(t), \phi_i, \theta_i) \\ &= \int_{K_i(t)}^{\infty} \frac{(1-\phi_i)\alpha}{\beta_1(\rho-\mu)}\left[\frac{\beta_1(\rho-\mu)}{(\beta_1-1)(1-\phi_i)\rho}\left(\frac{\rho\theta_i}{(1-\phi_i)\alpha\beta_1^{\alpha-1}} + w\right)\right]^{(1-\beta_1)} P(t)^{\beta_1} k_i^{(\alpha-1)\beta_1} dk_i \\ &\quad + K_i(t)^{\alpha}\left[\frac{(1-\phi_i)P(t)}{(\rho-\mu)} - \frac{w}{\rho}\right] - \theta_i K_i(t). \end{aligned} \tag{49}$$

Let us consider the problem in the first stage. If all other bidders report truthfully, bidder i chooses his report $\tilde{\theta}_i$ and exercise time τ_i to maximize his *interim utility*, which is calculated as follows:

$$\begin{aligned} &U_i(\theta_i, \tilde{\theta}_i, \tau_i) \\ &= \mathrm{E}_{\boldsymbol{\theta}_{-i}}\left\{ Q_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i}) \underset{\{K_i(\tau_i)\}}{\mathrm{Max}}\ \mathrm{E}\left[\int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i} K_i(\tau_i)^{\alpha}\left[(1-\phi_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i}))P(\tau_i) - w\right] d\tau_i \right.\right. \\ &\quad \left.\left. -\theta_i\left(1_{[dK_i(\tau_i)>0]}\right) dK_i(\tau_i)\right] - C_i(\tilde{\theta}_i, \boldsymbol{\theta}_{-i})\right\}. \end{aligned} \tag{50}$$

The equilibrium utility of bidder i, assuming that the other bidders report truthfully, is calculated as $T_i(\theta_i) = U_i(\theta_i, \theta_i, \tau_i^*(\theta_i, \phi_i(\theta_i, \boldsymbol{\theta}_{-i})))$. Incentive compatibility requires that $T_i(\theta_i) \geq U_i(\theta_i, \tilde{\theta}_i, \tau_i)$, and individual rationality requires that $T_i(\theta_i) \geq 0$.

By applying the revelation principle, we derive the optimal choice of $\tilde{\theta}_i$ as θ_i, and hence, bidder i 's equilibrium utility is calculated as

$$T_i(\theta_i) = \mathrm{E}_{\boldsymbol{\theta}_{-i}}\left\{\int_{\theta_i}^{\bar{\theta}} Q_i(\varepsilon, \boldsymbol{\theta}_{-i}) S_2(\varepsilon) d\varepsilon\right\} + T_i(\bar{\theta}), \tag{51}$$

where $S_2(\varepsilon) = \left[\underset{\{K_i(\tau_i^*)\}}{\mathrm{Max}} \mathrm{E}\int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i^*} 1_{[dK_i(\tau_i^*)>0]} dK_i(\tau_i^*)\right]$, and $\tau_i^* = \tau_i^*(\varepsilon, \phi_i(\varepsilon, \theta_{-i}))$. Taking expectations over bidder i's type θ_i and integrating Eq. (51) by parts yields bidder i's ex-ante utility before the auction takes place as follows:

$$\begin{aligned}
&\mathrm{E}_{\theta_i}[T_i(\theta_i)] \\
&= \mathrm{E}_{\boldsymbol{\theta}}\left\{Q_i(\boldsymbol{\theta})\left[\underset{\{K_i(\tau_i^*)\}}{\mathrm{Max}} \mathrm{E}\int_{\tau_{i0}}^{\infty} e^{-\rho\tau_i^*} 1_{[dK_i(\tau_i^*)>0]} dK_i(\tau_i^*) \frac{F(\theta_i)}{f(\theta_i)}\right]\right\} + T_i(\bar{\theta}).
\end{aligned} \tag{52}$$

The price of the incentive compatibility condition and the individual rationality conditions resembles their counterpartsthat for the case of lumpy investment.

We define welfare at $t = 0$ as the sum of the bidders' utilities and the seller's revenue at $t = 0$. Welfare, denoted by Y_2, thus equals the summation of the expected value of the option to invest net of the upfront cost which is expressed as follows:

$$Y_2 = \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\boldsymbol{\theta}) \mathrm{E}\left[e^{-\rho\tau_{i0}}(W_1(K_i(\tau_{i0}), P(\tau_{i0})) - \theta_i K_i(\tau_{i0}))\right]\right\}. \tag{53}$$

The seller's revenue in an auction with competitive bids at $t = 0$, denoted by R_2, is equal to the welfare as obtained using Eq. (53) minus the sum of all bidders' utilities as obtained using Eq. (52):

$$\begin{aligned}
R_2 &= \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\boldsymbol{\theta}) \mathrm{E}\left[e^{-\rho\tau_{i0}}(W_1(K_i(\tau_{i0}), P(\tau_{i0})) - \theta_i K_i(\tau_{i0}))\right] - \sum_{i=1}^{N} T_i(\theta_i)\right\} \\
&= \mathrm{E}_{\boldsymbol{\theta}}\left\{\sum_{i=1}^{N} Q_i(\boldsymbol{\theta}) \mathrm{E}\left[e^{-\rho\tau_{i0}}(W_1(K_i(\tau_{i0}), P(\tau_{i0})) - \left(\theta_i + \frac{F(\theta_i)}{f(\theta_i)}\right) K_i(\tau_{i0}))\right]\right\} - \sum_{i=1}^{N} T_i(\bar{\theta}),
\end{aligned} \tag{54}$$

where $\frac{F(\theta_i)}{f(\theta_i)} K_i(\tau_{i0})$ is the information rent received by bidder i at $t = \tau_{i0}$.

The seller chooses (Q_i, C_i, ϕ_i) to maximize the revenue obtained from Eq. (54) subject to the incentive compatibility condition and individual rationality, where bidder i chooses $\tau_i^*(\theta_i, \phi_i(\theta_i, \theta_{-i}))$ to maximize $U_i(\theta_i, \tilde{\theta}_i, \tau_i)$, as obtained using Eq. (50). Imposing the conditions of $w = 0$, $\tilde{\theta}_i = \theta_i$, and $\tau_i = \tau_i^*(\theta_i, \phi_i(\theta_i, \theta_{-i}))$ onto $U_i(\theta_i, \tilde{\theta}_i, \tau_i)$ reveals that bidder i chooses the exercise date to maximize R_2 if the following condition holds:

$$\frac{\phi_i P(t) K_i(t)^{\alpha}}{(\rho - \mu)} + \frac{\phi_i}{(1 - \phi_i)} \int_{K_i(t)}^{\infty} b_2 \theta_i^{1-\beta_1} P(t)^{\beta_1} K_i^{(\alpha-1)\beta_1} dK_i = \frac{F(\theta_i)}{f(\theta_i)} K_i(t), \tag{55}$$

where $b_2 = \left[\frac{\alpha(1-\phi_i)}{\beta_1(\rho-\mu)}\right]^{\beta_1} (\beta_1 - 1)^{(\beta_1-1)}$.

In equilibrium, the interim utility of bidder i, as expressed in Eq. (51), must be equal to his ex-post utility obtained from Eq. (50). The individual rationality condition requires that the ex-post utility of the least favorable type be equal to zero, $T_i(\bar{\theta}) = 0$. Using these two conditions and Eq. (55) and assuming that the perfect Bayesian Nash equilibrium holds, we determine that bidder i's cash payment in an FPA is obtained through $C_i''(\theta_i)$ in Eq. (20).

Q.E.D.

Appendix 5: Proof of Proposition 6

Differentiating ϕ_i'' in Eq. (19) with respect to $\alpha, \theta_i, \mu,\ \rho$, and σ and rearranging yields the following equations:

$$\frac{\partial \phi_i''}{\partial \alpha} = \frac{\Delta_1}{-\Delta} \gtrless 0, \tag{56}$$

$$\frac{\partial \phi_i''}{\partial \theta_i} = \frac{\Delta_2}{-\Delta} \gtrless 0, \tag{57}$$

$$\frac{\partial \phi_i''}{\partial \mu} = \frac{\Delta_3}{-\Delta} < 0, \tag{58}$$

$$\frac{\partial \phi_i''}{\partial \rho} = \frac{\Delta_4}{-\Delta} > 0, \tag{59}$$

and

$$\frac{\partial \phi_i''}{\partial \sigma} = \frac{\Delta_5}{-\Delta} < 0, \tag{60}$$

where

$$\Delta = -\left[\frac{\theta_i}{\alpha(1-\beta_1^{-1})(\phi_i''^{-1}-1)^2} + \frac{1}{(1-\beta_1^{-1})(\beta_1(1-\alpha)-1)(\phi_i''^{-1}-1)^2}\right]\frac{1}{\phi_i''^2} < 0, \tag{61}$$

$$\Delta_1 = \frac{\theta_i}{\alpha^2(\phi_i''^{-1}-1)(1-\beta_1^{-1})} - \frac{1}{(\phi_i''^{-1}-1)(1-\beta_1^{-1})(\beta_1(1-\alpha)-1)^2} \gtrless 0, \tag{62}$$

$$\Delta_2 = \left[\frac{\partial\left(F(\theta_i)/f(\theta_i)\right)}{\partial \theta_i} - \frac{1}{\alpha(\phi_i''^{-1}-1)(1-\beta_1^{-1})}\right] \gtrless 0 \tag{63}$$

$$\Delta_3 = E\frac{\partial \beta_1}{\partial \mu} < 0, \tag{64}$$

$$\Delta_4 = E\frac{\partial \beta_1}{\partial \rho} > 0, \tag{65}$$

$$\Delta_5 = E\frac{\partial\beta_1}{\partial\sigma} < 0, \quad (66)$$

and

$$E = \left\{ \frac{\theta_i}{\alpha(\phi_i''^{-1} - 1)(1 - \beta_1^{-1})^2\beta_1^2} + \frac{[2(\beta_1 - 1)(1 - \alpha) - 1]}{(\phi_i''^{-1} - 1)(\beta_1 - 1)^2[\beta_1(1 - \alpha) - 1]^2} \right\} > 0,$$

given that $\frac{\partial\beta_1}{\partial\mu} < 0$, $\frac{\partial\beta_1}{\partial\rho} > 0$, and $\frac{\partial\beta_1}{\partial\sigma} < 0$.**Q.E.D.**

Abbreviations

DOI	Department of the interior
FPA	First-price auction
OCS	Outer continental shelf

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Declarations

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The authors have no conflicts of interest to declare that are relevant to the content of this article.

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