RESEARCH ARTICLE



The interaction of emotions and cost-shifting rules in civil litigation

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Received: 14 November 2019 / Accepted: 16 March 2022 / Published online: 12 April 2022 © The Author(s) 2022, corrected publication 2022

Abstract

We model civil litigation as a simultaneous contest between a plaintiff and a defendant who have monetary and emotional preferences. The litigants' emotional variables capture a non-monetary joy of winning and relational emotions toward each other. A contest success function (CSF) describes the litigants' respective probabilities of success based on their endogenous litigation expenses and exogenous relative advantages. The model does not specify a functional form for the CSF. Instead, it accommodates any CSF that satisfies general and intuitive assumptions, which capture frequentlyused functional forms. A cost-shifting rule allows the winner to recover an exogenous proportion of her litigation expenses from the loser. There exists a unique Nash equilibrium with positive expenses. In equilibrium, negative relational emotions (but not a positive joy of winning) amplify the effects of cost shifting, and vice versa. Thus negative relational emotions and positive cost shifting have a similar strategic role, and one can be a substitute for the other. If the litigants' relative advantages are sufficiently balanced, then more cost shifting (or more negative relational emotions) increases total expenses in equilibrium.

Keywords Cost shifting \cdot Emotions \cdot Interdependent preferences \cdot Litigation \cdot Contest

JEL Classification C72 · D91 · K41

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1 Introduction

Civil litigation typically resolves contests between plaintiffs and defendants. Among the benefits of litigation are enforcement of substantive law, provision of guidance for future conduct and deterrence of future injuries (Shavell 1997). Litigation also generates enormous costs to the litigants and the society. These costs include the costs of hiring lawyers, discovering evidence, providing judges, and running courts. Moreover, some litigants incur legal costs that well exceed the monetary value of the prize of winning. For example, an American judge recalled a divorce case in which the husband spent millions just to keep the wife from having a painting that was sold for less than half a million (Duncan 2007 at p. 125). Inheritance disputes, especially between siblings or step-siblings, also tend to generate disproportionately high litigation costs (Vines 2011 at p. 27).

Contest theory is the standard tool for modeling litigation. A litigation model typically includes a contest success function (CSF) that maps the litigants' choices of litigation expenses to their respective probabilities of success. The existing models tend to assume that the CSF takes a specific functional form (see Katz and Sanchirico 2012). While the chosen functional form may appropriately capture some judicial systems in the real world, it may poorly capture others. It can be hard to ascertain which of the existing functional forms is "ideal" or closer to reality (compare Plott 1987 with Katz 1987, 1988; Farmer and Pecorino 1999; Carbonara et al. 2015; Dari-Mattiacci and Saraceno 2020 in Online Appendix D.2). For instance, selection bias arising from decisions to file suit or settle hinders efforts to empirically ascertain which one of the existing functional forms prevails. Moreover, while specifying a "nice" functional form can simplify the solution process, the resulting positive predictions and policy recommendations may not be robust to alternative functional forms (see Table 1). Litigation models should have robust theoretical foundations to ensure that their implications do not depend on the modelers' idiosyncrasies.

The existing litigation models also tend to assume that litigants are purely self interested (see Katz and Sanchirico 2012; Spier 2007 at pp. 300–305). However, contest experiments consistently suggest that subjects frequently consider relative

and non-monetary payoffs (see below). Litigation models may give rise to misleading predictions and policy recommendations if they neglect well-documented behavioral traits.

This paper develops contest theory to advance our understanding of litigation. In the proposed Emotional Litigation Game, the CSF satisfies general and intuitive assumptions instead of a specific functional form. The novel aspects of our Assumptions 1–8 are restrictions on the extent of interdependence in payoffs and on the curvature of the CSF. There is no assumption on cross-derivatives (compare with Friedman and Wickelgren 2013 at p. 506). Thus the players' strategies in our model are not generally strategic substitutes or complements. Assumptions 1–8 capture a large class of CSFs, including Tullock's (1980) and Plott's (1987) CSFs (see CSF (8) in Remark 1, CSF (10)).¹ Aside from generating novel results, the proposed model can facilitate verification of whether the results obtained in the literature remain valid under weaker assumptions.

Each litigant in the Emotional Litigation Game acts to maximize an emotional payoff that represents her expectations regarding her monetary outcome, her nonmonetary joy of winning and her negative or positive relational emotions toward the other litigant. The joy of winning arises from winning the lawsuit, while negative (respectively, positive) relational emotions arise from harming (benefiting) the adversary. We prove the existence and uniqueness of a pure-strategy Nash equilibrium with positive expenses.

We introduce spillovers (or externalities) to capture cost-shifting rules. These rules allocate litigation costs (including attorneys' fees) between the winner and loser of a lawsuit. Cost-shifting rules affect the litigants' strategic interaction and their incentives to spend.² On one end of the costs-shifting spectrum is the traditional American rule that requires each litigant to bear her own costs. On the other end is the idealized English rule that requires the loser to pay all of the winner's costs.³ Most judicial systems across the globe allow partial recovery (Katz and Sanchirico 2012 at pp. 273–275). The Emotional Litigation Game captures intermediate and extreme cost-shifting rules.

In the presence of cost shifting, the two different forms of non-monetary considerations—-relational emotions and a joy of winning—have different implications on equilibrium outcomes. Intuitively, a greater joy of winning directly increases a litigant's marginal benefits of spending. More negative relational emotions generate

¹ The Tullock CSF is widely applied in the litigation literature (Katz and Sanchirico 2012), and is consistent with the inference process of a Bayesian judge (Skaperdas and Vaidya 2012).

² By comparison, auction-theoretic models of settlement typically assume that once a case proceeds to trial, litigation expenses do not vary with the extent of cost shifting (for example, Bebchuk 1984 at p. 406; Klement and Neeman 2005 at p. 289; Klerman and Lee 2014 at pp. 216, 224; Landeo et al. 2006 at pp. 63, 74; Polinsky and Rubinfeld 1998 at p. 524; Spier 1994 at p. 200). Assuming fixed litigation expenses, Shavell (1997) and Spier (1997) proved that both the American rule and the English rule do not induce socially optimal amount of suits or settlement. Fixing litigation expenses in a generalized Nash bargaining model, Anderlini et al. (2019) proved the choice of cost-shifting rule does not affect whether suits are settled or litigated.

³ For contests in which the winner or loser(s) gets reimbursed by a contest designer, see Matros and Armanios (2009); Matros (2012). For contests in which a designer or principal chooses whether to disclose information, see Kaplan and Wettstein (2021); Serena (2021).

similar direct effects because the litigant has a heightened desire to harm her adversary. However, unlike the joy of winning, relational emotions have indirect effects in cases involving positive cost shifting. Positive cost shifting creates spillovers (in expectation) because, when a litigant chooses her expenses level, she expects that with a positive probability some or all of her costs are borne by her adversary. More negative relational emotions indirectly amplify such spillovers because the litigant derives a greater value from inflicting expected costs on her adversary. In fact, more negative relational emotions (or more cost shifting) heighten incentives to spend in an *asymmetric* manner; the litigant with stronger relative advantages experiences a *greater* increase in her incentive to spend, because her expected reward from doing so is greater than the weaker litigant's. Formalizing these observations, we prove that more negative relational emotions (or more cost shifting) increase the equilibrium relative expenses and probability of success in favor of the relatively more advantaged litigant (see Sect. 4).

Drastically different normative implications arise from the subtle differences between relational and outcome-dependent emotions. Our equilibrium analysis suggests that negative relational emotions amplify the cost-shifting rule, while a positive joy of winning has no such effect. Hence, to understand and optimize cost shifting, we need to take into account and respond differently to these two forms of emotions (see Sect. 6). For instance, while both, a positive joy of winning and negative relational emotions typically increase costs in litigated cases (see Sect. 5), only relational emotions interact with the cost-shifting rule.

Even if emotional concerns are absent, general formulations of cost-shifting rules and of CSFs give rise to novel insights on litigation spending. Call the sum of the litigants' expenses in a litigated case litigation expenditure. A well-known result in the literature is that in a litigated case, the English rule generates a greater litigation expenditure than the American rule does (for example, Braeutigam et al. 1984; Katz 1987; Plott 1987).⁴ This result may *not* carry over to intermediate cost-shifting rules and generally-formulated CSFs. In the equilibrium of the Emotional Litigation Game, if the litigants's relative advantages are sufficiently balanced (in a precise sense described in Sect. 4.3), then more cost shifting necessarily increases litigation expenditure.⁵ However, in cases involving extreme relative advantages, more cost shifting may or may not increase litigation expenditure; and imposition of participation constraints does not remove such ambiguity (see Remark 2). For instance, in these extremely one-sided cases, if the CSF takes the Tullock form (CSF (8) in Remark 1), and the American rule initially applies, then a small increase in cost shifting *decreases* litigation expenditure (see Sect. 4.3). The conventional wisdom that more cost shifting encourages spending in litigated cases is not robust to more general formulations of the cost-shifting rule.

 $^{^4}$ This result is supported by Fenn et al.'s (2017) natural experiment from the United Kingdom.

⁵ Baye et al. (2005) and Klemperer (2003) (in Appendix 1) proved a largely similar result with an auctiontheoretic model in which two symmetric litigants have private information and the highest spender wins with probability 1. While expected litigation expenses under the English rule are unbounded in their models, such expenses are bounded in our model. The present Assumption 7 ensures bounded expenses and, together with other assumptions, guarantees the existence and uniqueness of an equilibrium with positive and finite expenses.

Behavioral considerations, such as interdependent preferences, are well-documented in contests. As Millner and Pratt (1989) first observed and Dechenaux et al. (2015) (at pp. 614–616) recently surveyed, contest experiments consistently reveal that subjects often spend significantly greater than the equilibrium predictions of models based on pure self interest. An explanation is, in addition to the monetary outcome of winning, subjects frequently consider non-monetary and relative outcomes (for example, Mago et al. 2016; Price and Sheremeta 2011; Sheremeta 2010). Subjects' spite for their adversaries also can explain over-exertion (Herrmann and Orzen 2008; Fonseca 2009). Building upon Rabin (1993), Sano (2014) endogenously generated intentions-based reciprocity in symmetric Tullock contests. Alternatively, over-exertion in symmetric Tullock contests can reflect particular realizations of mixed strategies (Baye et al. 1999) or evolutionarily stable strategies consistent with contestants behaving to maximize relative payoff (Hehenkamp et al. 2004).

There is a small literature that explores the role of emotions in civil litigation. While several contributions studied pretrial bargaining and incentives to file suit or settle,⁶ Baumann and Friehe (2012) first introduced emotional variables into a Tullock model of litigation with endogenous expenses. Among their important findings was that introducing outcome-dependent emotions-an emotional gain from winning and an emotional loss from losing-has similar equilibrium implications as increasing the judgment sum in dispute (Baumann and Friehe 2012 at pp. 196, 203-204). Aside from capturing a whole class of CSFs, we complement their pioneering work in the following aspects: while they assumed no cost shifting, we permit any extreme or intermediate cost-shifting rule; while they only considered outcome-dependent emotions, we distinguish between relational emotions and outcome-dependent emotions (captured by our joy-of-winning parameter). Both additions are needed to reveal the strategic interaction of cost-shifting rules and relational emotions (see Proposition 2 in Sect. 3). However, unlike us, Baumann and Friehe (2012) (at. pp.196, 202-212) allowed the litigants to have asymmetric outcome-dependent emotions, and they used the specific functional forms of the Tullock CSF and of emotions to study incentives to sue and accuracy in adjudication. Moreover, our formulation of negative relational emotions is similar to Guha's (2019) formulation in a pretrial bargaining model with complete information. While we capture endogenous litigation expenses, Guha (2019) assumed exogenous litigation expenses and considered various bargaining procedures.

Building upon the seminal work of Skaperdas (1996), Clark and Riis (1998) offered axiomatic foundations for asymmetric Tullock CSFs. The assumptions stated by these authors permit *n*-players and require independence of irrelevant alternatives (IIA). Our Assumptions 1–8 restrict to two players but do not require IIA; Assumptions 1–8 thus capture some well-known CSFs that violate IIA (for example, Plott 1987; Beviá and Corchón 2015). Malueg and Yates (2006) also dropped IIA and found sufficient conditions for equilibrium existence and uniqueness. Their assumptions capture *n*-player symmetric contests while ours two-player asymmetric contests. Einy et al.

⁶ See Farmer and Tiefenthaler (2001) and the papers surveyed by Baumann and Friehe (2012) at pp. 197– 199. To our best knowledge, Huang and Wu (1992) first considered the effects of emotions on pretrial bargaining and decisions to bring suit or settle. Specifying extreme cost-shifting rules and exogenous litigation costs, Friehe and Pham (2021) studied how accident victims' intentions-based reciprocity affects settlement behavior and potential injurers' precautions.

(2015) proved equilibrium existence for Tullock contests with incomplete information but no spillovers. Ewerhart and Quartieri (2020) offered general sufficient conditions for equilibrium existence and uniqueness in Tullock and other logit-form contests with incomplete information but no spillovers. Haimanko (2021) established equilibrium existence for a general class of incomplete-information contests with possibly uncountable type-spaces but no spillovers. Our equilibrium existence and uniqueness result permits spillovers but assumes complete information. Moreover, Cornes and Hartley (2005) and others found that restricting the curvature of the CSF is crucial for guaranteeing equilibrium uniqueness in Tullock contests.⁷ Chowdhury and Sheremeta (2011b) found that failing to restrict the extent of spillovers in Tullock contests may lead to multiple equilibria. Imposing restrictions on both the curvature of the CSF and the spillovers (see Assumptions 5, 7), our equilibrium uniqueness result shows that their findings carry over to a large class of CSFs. Recent surveys of contest theory include Serena and Corchón (2017), Konrad (2009) and Vojnović (2016).

The present formulation of cost-shifting rules as proportions (of litigation expenses) resembles the formulation of linear spillovers by Chowdhury and Sheremeta (2011a; 2011b; 2015) in a two-player symmetric Tullock contest, Baye et al. (2012) in a two-player symmetric all-pay contest,⁸ and Petkov (2022) in an infinite-horizon multi-stage all-pay contest with two symmetric players. The formulation of spillovers by these authors is more general than ours, but we permit player asymmetry. Our formulation of cost-shifting rules is also the same as that adopted by Plott (1987), Gong and McAfee (2000) and Luppi and Parisi (2012),⁹ but the CSFs used by these authors assume symmetric players and take specific functional forms. Moreover, while strategies in Fu and Lu's (2013) two-player all-pay contest symmetrically affect the CSF but asymmetrically generate spillovers (as proportions), strategies in our model asymmetrically affect the CSF but symmetrically generate spillovers.

Section 2 constructs the Emotional Litigation Game. Section 3 establishes equilibrium existence and uniqueness. Sections 4, 5 conduct comparative statics analyses. Sections 6 offers a normative discussion and suggestions for future research. Appendix A contains all proofs. Appendix B provides calculations to facilitate presentation of examples.

2 The emotional litigation game

The Emotional Litigation Game is a simultaneous-move game of complete information characterized by two risk-neutral players—Plaintiff and Defendant—their common

⁷ Einy et al. (2020) proved that the unique Nash equilibrium of a complete-information Tullock contest (and of other games) is also the unique correlated equilibrium, which is robust to incomplete information in a strong sense.

⁸ Baye et al. (2012) also showed that spillovers can capture behavioral or evolutionary considerations.

⁹ Farmer and Pecorino (2016), Dari-Mattiacci and Saraceno (2020) (in Online Appendix D.2) and some others formulated a cost-shifting rule as an exogenous *quantity* below which the winner's costs are fully recoverable and above which her costs are fully unrecoverable. This quantity formulation is a special case of our proportion formulation. Carbonara et al. (2015) (at p.123) showed that the quantity formulation is strategically equivalent to extreme cost-shifting rules. Extreme cost-shifting rules are special cases of the proportion formulation.

set of actions \mathbb{R}_+ , and their payoff functions described below. Each payoff function has monetary and non-monetary components, including a joy of winning and the player's emotions toward her adversary. The payoff functions and exogenous parameters are common knowledge.

Plaintiff and Defendant simultaneously choose e_P , $e_D \ge 0$ levels of expenses, respectively.¹⁰ An exogenous parameter $0 < \mu < 1$ represents Plaintiff's relative advantages; $1 - \mu$ represents Defendant's relative advantages. Plaintiff (respectively, Defendant) is relatively more advantaged if $\mu > 0.5$ ($\mu < 0.5$). Relative advantages capture institutional factors that do not vary with litigation expenses but influence the outcome of the case.¹¹ The judicial process with probability $\theta(e_P, e_D; \mu)$ requires Defendant to transfer a judgment sum 1 to Plaintiff, where the contest success function (CSF) $\theta : \mathbb{R}^2_+ \rightarrow [0, 1]$ is continuous in $\mathbb{R}^2_+ \setminus \{(0, 0)\}$ and twice continuously differentiable in \mathbb{R}^2_{++} . Upon determination of the outcome of the case, a cost-shifting rule requires the loser to pay an exogenous $0 \le \hat{\lambda} \le 1$ proportion of the winner's costs. In particular, $\hat{\lambda} = 0$ characterizes the American rule that requires no recovery, and $\hat{\lambda} = 1$ the English rule that allows for full recovery. Containing the monetary variables are Plaintiff's and Defendant's respective monetary payoffs u_P , $u_D : \mathbb{R}^2_+ \rightarrow \mathbb{R}$ given by

$$u_P(e_P, e_D) = \theta u_P(e_P, e_D | \text{Plaintiff wins}) + (1 - \theta) u_P(e_P, e_D | \text{Defendant wins})$$
(1)

 $u_D(e_P, e_D) = \theta u_D(e_P, e_D | \text{Plaintiff wins}) + (1 - \theta) u_D(e_P, e_D | \text{Defendant wins})$ (2)

where

$$\begin{cases} u_P(e_P, e_D | \text{Plaintiff wins}) = 1 - (1 - \widehat{\lambda})e_P \\ u_P(e_P, e_D | \text{Defendant wins}) = -e_P - \widehat{\lambda}e_D \\ \begin{cases} u_D(e_P, e_D | \text{Plaintiff wins}) = -1 - e_D - \widehat{\lambda}e_P \\ u_D(e_P, e_D | \text{Defendant wins}) = -(1 - \widehat{\lambda})e_D. \end{cases}$$
(3)

Each litigant's monetary payoff reflects her expected monetary outcome. If Plaintiff wins, her monetary payoff equals the judgment sum 1 less the unrecoverable proportion of her expenses, $(1 - \hat{\lambda})e_P$. If Plaintiff loses, she pays all her expenses e_p and the recoverable proportion of Defendant's expenses, $\hat{\lambda}e_D$. The weights θ , $1 - \theta$ are respectively Plaintiff's probabilities of winning and losing. Defendant's monetary payoff has a similar interpretation.

¹⁰ This specification assumes the litigants have sufficiently large budgets, so their best replies are the result of unconstrained optimization (see the proof of Proposition 1 in Appendix A). Budget constraints may potentially give contestants incentives to settle their contests by making side-payments. Compare Beviá and Corchón (2010) with Kimbrough and Sheremeta (2013).

¹¹ These institutional factors may reflect the inherent merits of the case (see Sect. 4.4), or judicial consideration of salient but legally irrelevant features of the case (for example, Bordalo et al. 2015). The judge (who is not an explicit player) may also rely on her personal and professional experiences, and may have her own biases and policy preferences (for example, Gennaioli and Shleifer 2007, 2008).

In addition to any monetary transfer, the winner derives an exogenous value $v \ge 0$, called the joy of winning. Moreover, each litigant derives value from her feelings about the other litigant's outcome; an exogenous $\xi < 1$ captures such relational emotions, meaning that each litigant is indifferent between one unit of her own monetary payoff (or joy of winning) and ξ^{-1} units of her adversary's. Containing the monetary and non-monetary variables are Plaintiff's and Defendant's respective emotional payoffs $\hat{u}_P, \hat{u}_D : \mathbb{R}^2_+ \to \mathbb{R}$ given by

$$\widehat{u}_P(e_P, e_D) = \theta \widehat{u}_P(e_P, e_D | \text{Plaintiff wins}) + (1 - \theta) \widehat{u}_P(e_P, e_D | \text{Defendant wins})$$
(4)

$$\widehat{u}_D(e_P, e_D) = \theta \widehat{u}_D(e_P, e_D | \text{Plaintiff wins}) + (1 - \theta) \widehat{u}_D(e_P, e_D | \text{Defendant wins})$$
(5)

where

$$\begin{cases} \widehat{u}_{P}(e_{P}, e_{D} | \text{Plaintiff wins}) = 1 - (1 - \widehat{\lambda})e_{P} + \upsilon + \xi[-1 - e_{D} - \widehat{\lambda}e_{P}] \\ \widehat{u}_{P}(e_{P}, e_{D} | \text{Defendant wins}) = -e_{P} - \widehat{\lambda}e_{D} + \xi[-(1 - \widehat{\lambda})e_{D} + \upsilon] \\ \end{cases} \\\begin{cases} \widehat{u}_{D}(e_{P}, e_{D} | \text{Plaintiff wins}) = -1 - e_{D} - \widehat{\lambda}e_{P} + \xi[1 - (1 - \widehat{\lambda})e_{P} + \upsilon] \\ \widehat{u}_{D}(e_{P}, e_{D} | \text{Defendant wins}) = -(1 - \widehat{\lambda})e_{D} + \upsilon + \xi[-e_{P} - \widehat{\lambda}e_{D}]. \end{cases}$$
(6)

Each litigant's emotional payoff contains monetary and emotional variables. If Plaintiff wins, then her emotional payoff includes her monetary outcome (the first two terms), the joy of winning v, and her relational emotions regarding Defendant's monetary outcome, $\xi[-1 - e_D - \hat{\lambda}e_P]$. If Plaintiff loses, then her emotional payoff includes her monetary outcome (the first two terms), and her relational emotions regarding Defendant's monetary outcome and joy of winning, $\xi[-(1 - \hat{\lambda})e_D + v]$. The weights θ , $1 - \theta$ are respectively Plaintiff's probabilities of winning and losing. Defendant's emotional payoff \hat{u}_D has a similar interpretation.

While the present formulation of relational emotions permits $0 < \xi < 1$, the empirical literature on contests suggests that it is typical to have $\xi < 0$ (see Sect. 1). We exclude the possibility that each litigant values her adversary's monetary payoff (or joy of winning) more than her own; that is, we rule out $\xi \ge 1$. Some algebra using the litigants' emotional payoffs (4), (5) will reveal that, in the limiting case of $\xi = 1$, each litigant would be incentivized to minimize total litigation expenses; she would only spend zero in any equilibrium.

Each litigant acts to maximize her emotional payoff. The exogenous parameters and payoff functions are common knowledge between the litigants. Assume there is no settlement or risk of default.¹²

¹² Litigants who settle do so in the shadow of the law; that is, they reach a settlement with an expectation of what the outcome would have been if their case had been adjudicated upon. Studying litigation outcomes in the absence of settlement is thus essential to understanding settlement negotiation. The litigants' equilibrium expenses in the present Emotional Litigation Game are the surplus to be shared in a pre-game that models their settlement negotiation. The litigants' equilibrium payoffs in litigation are their outside options in that pre-game. This paper thus provides the parameters for future research projects that comprehensively study settlement negotiation (see Sect. 6).

We now state and impose the following Assumptions 1–8 to ensure interior equilibrium existence and uniqueness.

Assumption 1 Fixing litigation expenses, the litigants are symmetric except in respect of their relative advantages. Formally, $\theta(e_1, e_2; \mu_0) = 1 - \theta(e_2, e_1; 1 - \mu_0)$, for any $e_1, e_2 \ge 0$ and any $0 < \mu_0 < 1$.

Assumption 1 requires the relative-advantages parameter μ to capture any asymmetry between the litigants that does not vary with their expenses. The litigants are symmetric if and only if $\mu = 0.5$.

Assumption 2 Holding relative advantages constant, proportionate changes in expenses levels do not affect Plaintiff's probability of success. Formally, $\theta(e_P, e_D; \mu) = \theta(xe_P, xe_D; \mu)$ for all x > 0 and all $(e_P, e_D) \in \mathbb{R}^2_+ \setminus \{(0, 0)\}$.

Assumption 3 Holding the litigation expenses constant, Plaintiff's probability of success is strictly increasing with her relative advantages. Formally, $\frac{\partial \theta}{\partial \mu} > 0$ for all $(e_P, e_D) \in \mathbb{R}^2_{++}$.

Assumption 4 Holding relative advantages and Defendant's expenses constant, Plaintiff's probability of success is strictly increasing with her expenses. Formally, $\frac{\partial \theta}{\partial e_P} > 0$ for all $(e_P, e_D) \in \mathbb{R}^2_{++}$.

Assumptions 2–4 capture intuitions regarding the properties of reasonable CSFs. Under Assumption 2, proportionate changes in expenses levels do not vary Plaintiff's probability of success. Assumption 3 requires that holding all else constant, an increase in Plaintiff's relative advantages strictly improves her probability of success. Assumption 4 requires that holding all else constant, Plaintiff is more likely to win if she spends more and Defendant incurs positive expenses. An implication of Assumption 4 is that Plaintiff does not win with probability 1 when both litigants incur positive and finite expenses.

Assumption 5 restricts the combined "strength" of the cost-shifting rule $\hat{\lambda}$ and relational emotions ξ . Define a proportion $\lambda = \hat{\lambda}(1 - \xi)$ and call it the emotionally-scaled cost-shifting rule. Such cost-shifting rule is scaled up (respectively, down) by negative (positive) relational emotions.

Assumption 5 The emotionally-scaled cost-shifting rule satisfies $\lambda \leq 1$.

Assumption 5 restricts the extent of interdependence in payoffs. Such restriction only matters in cases of negative relational emotions ($\xi < 0$). As the system of first order conditions (20) in Appendix A will formalize, Assumption 5 ensures that each litigant's marginal costs of spending are always positive; allowing for $\lambda > 1$ might induce zero marginal costs for some extremely asymmetric expenses pairs.

Assumption 6 Suppose the emotionally-scaled cost-shifting rule satisfies $\lambda = 1$. Then Plaintiff does not win with probability 1 by spending infinitely more than Defendant does. Formally, if $\lambda = 1$, then $\lim_{e_P/e_D \to +\infty} \theta < 1$.

Assumption 6 applies only in cases involving "strong" negative relational emotions and cost shifting (in the precise sense of satisfying $\lambda = 1$). In these cases, Assumption 6 ensures that Plaintiff does not have an incentive to make explosive relative expenses $(e_P/e_D \rightarrow +\infty)$ with the expectation that she almost surely passes her expenses onto Defendant and derives an infinitely large value from doing so. Similarly, Assumptions 1 and 6 together imply Defendant does not win almost surely by spending infinitely more than Plaintiff does;¹³ formally, if $\lambda = 1$, then $\lim_{e_D/e_P \rightarrow +\infty} \theta > 0$.

Assumption 7 For all $(e_P, e_D) \in \mathbb{R}^2_{++}$, the parameters ξ , $\hat{\lambda}$ and the CSF $\theta(\cdot)$ together satisfy

$$\frac{\frac{\partial^2}{\partial e_P^2} \left(\frac{\theta}{1-\lambda\theta}\right)}{\frac{\partial}{\partial e_P} \left(\frac{\theta}{1-\lambda\theta}\right)} < 0.$$
(7)

Assumption 7 restricts the CSF $\theta(\cdot)$ and the parameters ξ , $\hat{\lambda}$ collectively. The lefthand side of inequality (7) describes the curvature of the ratio $\theta/(1 - \lambda\theta)$, which lies between Plaintiff's probability of success θ and her relative probability of success $\theta/(1 - \theta)$; that is, $\theta \le \theta/(1 - \lambda\theta) \le \theta/(1 - \theta)$. Distorted by the emotionally-scaled cost-shifting rule λ , which depends on ξ and $\hat{\lambda}$, the ratio $\theta/(1 - \lambda\theta)$ approaches θ (respectively, $\theta/(1 - \theta)$) as λ approaches 0 (respectively, 1). Requiring the curvature of $\theta/(1 - \lambda\theta)$ to be negative, Assumption 7 ensures that Plaintiff's payoff function is strictly quasiconcave in her own expenses (see the proof of Lemma 3 in Appendix A).

Intuitively, Assumptions 5–7 eliminate incentives to incur explosive expenses. Such incentives may arise in cases where the litigants are motivated by "too much" negative relational emotions ξ and cost shifting $\hat{\lambda}$, which "overshadow" monetary costs and the judgment sum in dispute. To guarantee equilibrium existence, Assumption 5 rules out cases where the combined "strength" of $\hat{\lambda}$ and ξ is "too high" for any CSF. Assumptions 6–7 further eliminate cases where the combined "strength" of $\hat{\lambda}$ and ξ is "too high" of $\hat{\lambda}$ and ξ is "too high" given the properties of the particular CSF that applies.

Assumption 8 Suppose Defendant incurs zero expenses. Then Plaintiff has a greater of probability of success if she incurs positive expenses than zero expenses. Formally, $\theta(e_P, 0) > \theta(0, 0)$ for all $e_P > 0$.

Assumption 8 is introduced to rule out any Nash equilibrium in which a litigant incurs zero expenses (see Proposition 3 below). We do not require the CSF to be continuous at the origin. Among the functional forms that satisfy Assumption 8 is the popular Tullock form (see CSF (8) below), which is discontinuous at the origin. A consequence of Assumption 8 is that if both litigants incur zero expenses, then both win with positive probabilities.

The solution concept adopted is an interior Nash equilibrium that comprises positive strategies by both litigants.

¹³ Part 4 of Lemma 2 in Appendix A formally proves this implication.

Remark 1 Fix a constant $\gamma > 0$. The existing litigation models typically apply the Tullock CSF $\theta_T : \mathbb{R}^2_+ \to [0, 1]$ given by:

$$\theta_T(e_P, e_D) = \begin{cases} \frac{\mu e_P^{\gamma}}{\mu e_P^{\gamma} + (1-\mu)e_D^{\gamma}} & \text{if } e_P + e_D > 0, \\ \mu & \text{otherwise.} \end{cases}$$
(8)

Specifying θ_T and no emotions, Farmer and Pecorino (1999) (pp. 281–282) and Carbonara et al. 2015 (pp. 126–127) showed that the English rule ($\hat{\lambda} = 1$) induces an interior Nash equilibrium if and only if the exponent $\gamma < 1$. Their conditions for equilibrium existence are special cases of Assumption 7 of the present Emotional Litigation Game. Assume relational emotions $\xi = 0$. Use Appendix B and the chain rule to obtain

$$\frac{\frac{\partial^2}{\partial e_P^2} \left(\frac{\theta_T}{1-\widehat{\lambda}\theta_T}\right)}{\frac{\partial}{\partial e_P} \left(\frac{\theta_T}{1-\widehat{\lambda}\theta_T}\right)} = \gamma e_P^{\gamma-1} \left[\frac{\gamma-1}{e_P} - \frac{2\mu(1-\widehat{\lambda})}{(1-\widehat{\lambda}\theta_T)[\mu e_P^{\gamma} + (1-\mu)e_D^{\gamma}]}\right]$$

which is strictly negative in cases where $\gamma \leq 1$ and $\hat{\lambda} < 1$, or $\gamma < 1$ and $\hat{\lambda} \leq 1$. These cases satisfy Assumption 7 as well as Assumptions 1–6, 8. Proposition 1 (stated in Sect. 3) will prove the existence and uniqueness of an interior Nash equilibrium. There is no such equilibrium if $\hat{\lambda} = 1$ and $\gamma = 1$; these specifications violate Assumption 7.

3 Equilibrium

This section establishes equilibrium existence and uniqueness. To facilitate a comparative statics analysis, we further prove the Emotional Litigation Game is strategically equivalent to a monetary game with a scaled cost-shifting rule.

Lemma 1 finds a unique, positive expenses ratio which will be used to characterize the interior Nash equilibrium. To simplify notation, define an auxiliary variable $s = e_D/e_P$ whenever Plaintiff's expenses level $e_P > 0$; s is the ratio of Defendant's expenses relative to Plaintiff's. Assumption 2 implies that for any two pairs of positive expenses $(e_P, e_D), (e'_P, e'_D) \in \mathbb{R}^2_{++}$ satisfying $e_D/e_P = e'_D/e'_P$, the CSF satisfies $\theta(e_P, e_D) = \theta(e'_P, e'_D)$. By a slight abuse of notation, denote $\theta(s) = \theta(e_P, e_D)$ and $\theta_s = \frac{\partial}{\partial s} \theta(s)$. Appendix A contains all proofs.

Lemma 1 There exists a unique positive expenses ratio $0 < s^* < +\infty$ that satisfies

$$s^* = \frac{1 - \widehat{\lambda}(1 - \xi)\theta(s^*)}{1 - \widehat{\lambda}(1 - \xi)[1 - \theta(s^*)]}.$$

The value of s^{*} *satisfies the following properties:*

1. If the American rule applies $(\widehat{\lambda} = 0)$, then $s^* = 1$.

2. If the cost-shifting rule allows positive recovery ($\hat{\lambda} > 0$), and Plaintiff's relative advantages satisfy $\mu > 0.5$ (respectively, = 0.5, < 0.5), then $s^* < 1$ (respectively, = 1, > 1).

Proposition 1 establishes the existence and uniqueness of an interior Nash equilibrium, and characterizes it.

Proposition 1 There exists a unique interior Nash equilibrium $(e_P^*, e_D^*) \in \mathbb{R}^2_{++}$, which is characterized by

$$e_P^* = \frac{-(1+\upsilon)(1-\xi)\theta_s(s^*)}{1-\widehat{\lambda}(1-\xi)(1-\theta(s^*)) + \widehat{\lambda}(1-\xi)(1+s^*)\theta_s(s^*)} \qquad e_D^* = s^* e_P^*$$

where Lemma 1 gives s*.

This Nash equilibrium satisfies the following properties:

- 1. If the American rule applies or the litigants' relative advantages are equal, then their expenses are equal. Formally, $\hat{\lambda} = 0$ or $\mu = 0.5$ implies $e_P^* = e_D^*$. Moreover, equal relative advantages imply equal probabilities of success. Formally, $\mu = 0.5$ implies $\theta(s^*) = 0.5$.
- 2. If the cost-shifting rule allows positive recovery, then the relatively more advantaged litigant spends relatively more and has a relatively greater probability of success. Formally, $\hat{\lambda} > 0$ and $\mu > 0.5$ (respectively, $\mu < 0.5$) implies $e_P^* > e_D^*$ and $\theta(s^*) > 0.5$ ($e_P^* < e_D^*$ and $\theta(s^*) < 0.5$).

Proposition 1 finds and characterizes the unique interior Nash equilibrium of the Emotional Litigation Game. All subsequent discussions of the Game's equilibrium refer to this interior Nash equilibrium. Although the expressions for the equilibrium expenses are complicated, application of the American rule ($\hat{\lambda} = 0$) leads to equal equilibrium expenses ($s^* = 1$). Under other cost-shifting rules, $s^* = 1$ also holds in the limit when relational emotions $\xi \to 1$.

Proposition 2 will clarify the exact roles that the emotional parameters play in equilibrium. For motivation, rearrange the components (6) of Plaintiff's emotional payoff \hat{u}_P (4) given the cost-shifting rule $\hat{\lambda}$, and the components (3) of her monetary payoff \hat{u}_P (1) given a potentially different cost-shifting rule $0 \le \hat{\lambda}' \le 1$, as follows:

$$\begin{aligned} \widehat{u}_{P}(e_{P}, e_{D} | \text{Plaintiff wins}) &= 1 + \upsilon - \xi + [\widehat{\lambda}(1 - \xi) - 1]e_{P} - \xi e_{D} \\ \widehat{u}_{P}(e_{P}, e_{D} | \text{Defendant wins}) &= \upsilon \xi - e_{P} - [\xi + \widehat{\lambda}(1 - \xi)]e_{D}, \\ u_{P}(e_{P}, e_{D} | \text{Plaintiff wins}) &= 1 + (\widehat{\lambda}' - 1)e_{P} \\ u_{P}(e_{P}, e_{D} | \text{Defendant wins}) &= -e_{P} - \widehat{\lambda}'e_{D}. \end{aligned}$$

An examination of \hat{u}_P reveals that Plaintiff's marginal benefits of winning sum to $(1 + \upsilon)(1 - \xi) + \hat{\lambda}(1 - \xi)(e_P + e_D)$; the second term is linear in aggregate expenses. An examination of u_P obtains Plaintiff's marginal monetary benefits of winning as $1 + \hat{\lambda}'(e_P + e_D)$; the second term is again linear in aggregate expenses. Plaintiff's payoff structure when she is motivated by both monetary and emotional variables resembles her payoff structure if she were only motivated by *appropriately scaled* monetary variables. A similar observation applies to Defendant.

To simplify presentation, fix and suppress the relative-advantages parameter μ . Let $\mathbb{G}(\xi, \upsilon, \lambda)$ denote the Emotional Litigation Game when a generic triple ξ, υ, λ respectively capture the relational emotions, joy of winning, and cost-shifting rule. Let $e_P^*(\xi, \upsilon, \lambda)$ and $\theta^*(\xi, \upsilon, \lambda)$ respectively denote Plaintiff's equilibrium expenses and probability of success in $\mathbb{G}(\xi, \upsilon, \lambda)$. Similarly, let $e_D^*(\xi, \upsilon, \lambda)$ and $s^*(\xi, \upsilon, \lambda) = e_D^*(\xi, \upsilon, \lambda)/e_P^*(\xi, \upsilon, \lambda)$ respectively denote Defendant's equilibrium (absolute) expenses and expenses relative to Plaintiff's. In the special case of $\upsilon = \xi = 0$, call the game a Monetary Litigation Game, because the litigants act only to maximize their monetary payoffs.

Proposition 2 The Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ is strategically equivalent to the Monetary Litigation Game $\mathbb{G}(0, 0, \lambda)$ in the following sense. Consider the equilibrium $(e_p^*(\xi, \upsilon, \widehat{\lambda}), e_D^*(\xi, \upsilon, \widehat{\lambda}))$ of the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ and the equilibrium $(e_p^*(0, 0, \lambda), e_D^*(0, 0, \lambda))$ of the Monetary Litigation Game $\mathbb{G}(0, 0, \lambda)$. Each litigant's equilibrium expenses level in $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ is $(1 + \upsilon)(1 - \xi)$ times her equilibrium expenses level in $\mathbb{G}(0, 0, \lambda)$. These two Games have the same equilibrium relative expenses and probabilities of success. Formally, $e_p^*(\xi, \upsilon, \widehat{\lambda}) = (1 + \upsilon)(1 - \xi)e_p^*(0, 0, \lambda)$, $e_D^*(\xi, \upsilon, \widehat{\lambda}) = (1 + \upsilon)(1 - \xi)e_D^*(0, 0, \lambda)$, $s^*(\xi, \upsilon, \widehat{\lambda}) = s^*(0, 0, \lambda)$, and $\theta^*(\xi, \upsilon, \widehat{\lambda}) = \theta^*(0, 0, \lambda)$.

Proposition 2 reveals a bijective relationship between the equilibrium expenses in the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ and those in the Monetary Litigation Game $\mathbb{G}(0, 0, \lambda)$. The emotionally-scaled cost-shifting rule $\lambda = \widehat{\lambda}(1 - \xi)$ adopted in $\mathbb{G}(0, 0, \lambda)$ is generally not the same as the unscaled cost-shifting rule $\widehat{\lambda}$ adopted in $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$.

Relational emotions have direct and indirect effects on equilibrium expenses, while the joy of winning only has direct effects. Suppose relational emotions are negative in $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$; that is, $\xi < 0$. Indirectly, such negative relational emotions render each litigant's equilibrium expenses in $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ to be proportionate to her equilibrium expenses in $\mathbb{G}(0, 0, \lambda)$, in which the cost-shifting rule is scaled up by $(1-\xi)$. Directly, each litigant's equilibrium expenses in $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$ is also scaled up by $(1-\xi)$ compared to her equilibrium expenses in $\mathbb{G}(0, 0, \lambda)$. (The opposite direct and indirect effects arise if relational emotions are positive, $0 < \xi < 1$.) By comparison, a positive joy of winning $\upsilon > 0$ only directly scales up equilibrium expenses by $(1 + \upsilon)$; υ does *not* (indirectly) affect relative expenses in equilibrium.

Unlike the joy of winning, relational emotions modify the effects of cost shifting. In particular, equilibrium expenses given negative relational emotions ($\xi < 0$) and the cost-shifting rule $\hat{\lambda}$ are enlargements of equilibrium expenses given pure self interest ($\xi = 0$) and a greater cost-shifting rule λ . In this sense, negative relational emotions are a substitute for a high-powered cost-shifting rule, and vice versa. The opposite is true in respect of positive relational emotions, $0 < \xi < 1$, which "weaken" the cost-shifting rule. Because it is typical to have negative relational emotions $\xi < 0$ in litigation (see Sect. 1), the "true" effects of cost shifting are greater than what they would have been if the litigants were purely self-interested. Section 6 will explore potential normative implications.

To facilitate subsequent comparative statics analysis, Corollary 1 ensures equilibrium existence when the parameters vary. Let $\Xi \subset (-\infty, 1)$ denote a set of relational-emotions parameters. Let $\Lambda \subset [0, 1]$ denote a set of cost-shifting parameters. Let $\Theta(\Xi, \Lambda)$ denote the set of CSFs $\theta : \mathbb{R}^2_+ \to [0, 1]$ that are continuous in $\mathbb{R}^2_+ \setminus \{(0, 0)\}$, twice continuously differentiable in \mathbb{R}^2_{++} , and satisfy Assumptions 1–8 for all $(\xi, \widehat{\lambda}) \in \Xi \times \Lambda$.

Corollary 1 Consider a $CSF \ \theta \in \Theta(\{\xi_0\}, \{\widehat{\lambda}_0\})$ and a pair $\xi_0, \widehat{\lambda}_0$ characterizing the relational emotions and cost-shifting rule, respectively. Given the same θ and a possibly different pair $\xi, \widehat{\lambda}$ that satisfies $\widehat{\lambda}(1-\xi) \leq \widehat{\lambda}_0(1-\xi_0)$, the Emotional Litigation Game also satisfies Assumptions 1–8 and therefore has a unique interior Nash equilibrium.

Given a CSF $\theta \in \Theta(\{\xi_0\}, \{\widehat{\lambda}_0\})$ where ξ_0 characterizes some initial relational emotions and $\widehat{\lambda}_0$ some initial cost-shifting rule, Corollary 1 guarantees interior equilibrium existence and uniqueness under any alternative pair $\xi, \widehat{\lambda}$ satisfying $\widehat{\lambda}(1-\xi) \leq \widehat{\lambda}_0(1-\xi_0)$. From now, we will analyze variations in the parameters to the extent that such variations are within the scope of Corollary 1.

Proposition 3 There is no Nash equilibrium in which one or more litigant incurs zero expenses. In particular, $(e_P, e_D) = (0, 0)$ is not a Nash equilibrium.

Proposition 3 establishes that non-interior Nash equilibrium does not exist in the Emotional Litigation Game. The interior equilibrium existence and uniqueness result in Proposition 1 only requires Assumptions 1–7. The addition of Assumption 8 implies that if a litigant incurs zero expenses, then the other litigant has no best reply.¹⁴ This implication leads to the non-existence result in Proposition 3. Proposition 3 thus allows subsequent comparative statics analysis to focus on the unique interior equilibrium given by Proposition 1.

4 Variations in non-emotional parameters

This section considers how variations in the non-emotional parameters affect equilibrium outcomes. We analyze the non-emotional parameters separately from the emotional parameters (see Sect. 5), because civil lawsuits in reality differ according to the role of emotions. For example, contract disputes between firms tend to be driven by monetary concerns only (Schwartz and Scott 2009 at pp. 947–948). Our comparative statics analysis of the emotional parameters is unlikely to offer insights for these commercial disputes. However, such analysis of the emotional parameters can offer insights for disputes concerning inheritance or divorce, which tend to involve strong emotions (see Sects. 1, 6). While this section's analysis of the non-emotional parameters is relevant for most civil lawsuits, the analysis of the emotional parameters in Sect. 5 only matters for lawsuits involving emotionally-charged litigants.

¹⁴ See the proof of Proposition 3 in Appendix A.

For simplicity, this section will analyse the Monetary Litigation Game $\mathbb{G}(0, 0, \lambda)$ with the emotionally-scaled cost-shifting rule λ . Due to Proposition 2, the same analysis applies to the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \hat{\lambda})$ with the unscaled cost-shifting rule $\hat{\lambda}$.

4.1 Non-emotional variables, relative expenses and probabilities of success

Corollary 2 Consider the equilibrium of the Monetary Litigation Game.

- 1. If the litigants' relative advantages are equal, then their relative expenses and probabilities of success do not vary with the cost-shifting rule. Formally, $\mu = 0.5$ implies $\frac{ds^*}{d\lambda} = 0$, $\frac{d\theta^*}{d\lambda} = 0$.
- 2. If Plaintiff (respectively, Defendant) is relatively more advantaged, then more cost shifting increases her expenses relative to Defendant's (Plaintiff's) and her probability of success. Formally, $\mu > 0.5$ ($\mu < 0.5$) implies $\frac{ds^*}{d\lambda} < 0$, $\frac{d\theta^*}{d\lambda} > 0$ ($\frac{d\theta^*}{d\lambda} > 0$, $\frac{d\theta^*}{d\lambda} < 0$).

Corollary 2 proves that the litigants' relative advantages determine how their equilibrium relative expenses and probabilities of success respond to variations in the extent of cost shifting. Intuitively, when relative advantages are not equal, more cost shifting incentivizes the relatively more advantaged litigant—who has better prospects of winning—to spend relatively more than the weaker litigant does. Then both relative advantages and relative expenses are in favor of the relatively more advantaged litigant; a greater equilibrium probability of success for her follows. Farmer and Pecorino (1999) (at p. 281), Carbonara et al. (2015) (at p. 126) analyzed the extreme American and English rules with a Tullock model, and obtained similar results. The present Corollary 2 confirms these results for generally-formulated CSFs and intermediate cost-shifting rules.

Corollary 3 Consider the equilibrium of the Monetary Litigation Game.

- If the American rule applies, then an increase in a litigant's relative advantages does not affect her relative expenses, but increases her probability of success. Formally, λ = 0 implies ds*/dμ = 0 and dθ*/dμ > 0.
 If the cost-shifting rule allows positive recovery, then a litigant's expenses relative
- 2. If the cost-shifting rule allows positive recovery, then a litigant's expenses relative her adversary's and her probability of success increase with with her relative advantages. Formally, $\lambda > 0$ implies $\frac{ds^*}{d\mu} < 0$ and $\frac{d\theta^*}{d\mu} > 0$.

Corollary 3 proves that the equilibrium implications of changes in a litigant's relative advantages depend on the cost-shifting rule. Part 1 proves that under the American rule $(\lambda = 0)$, becoming more advantaged does not incentivize a litigant to spend relatively more in equilibrium. Nonetheless, the increase in her relative advantages has a direct effect that improves her equilibrium probability of success.¹⁵ Part 2 proves that if positive cost shifting takes place ($\lambda > 0$), becoming more advantaged incentivizes the litigant to spend relatively more in equilibrium. Then, in addition to the direct effect, the increase in her relative advantages indirectly improves her equilibrium probability of success through increasing relative expenses in her favor.

¹⁵ For a similar result in a two-player general contest with no spillovers, see Levine and Mattozzi's (2021) analysis of whether lower cost of effort leads to greater success across equilibria and CSFs.

4.2 Relative advantages and expenditure

A well-known result in the Tullock contests literature is that a more asymmetric contest reduces rent dissipation (for example, Cornes and Hartley 2005 at pp. 940–941). In litigation models that use the Tullock CSF given by (8), this result implies that in a case that proceeds to litigation, total litigation expenses decrease when the case becomes more one-sided (for example, Carbonara et al. 2015 at p. 121). This section shows that this result is not robust to more general formulations of the CSF.

Let \mathbb{E}^* denote the sum of the litigants' expenses in equilibrium, and call it litigation expenditure:

$$\mathbb{E}^* = e_P^* + e_D^*.$$
 (9)

This definition of litigation expenditure only captures the litigation expenses borne by those litigants who proceed to litigation. Excluded from calculation are the costs borne by the judicial system or the society. The definition of \mathbb{E}^* also does not capture how private or public litigation expenses change in response to decisions to bring suit, defend suit, or settle (see Remark 2, Sect. 6). A comprehensive (and complex) model that includes the society's perspective on the costs and benefits of litigation is required to resolve issues regarding the optimal balance between the litigants' private interests and the interests of the society. These issues cannot be resolved without a comprehensive and robust analysis of how cost-shifting rules affect private litigation expenditure in litigated cases.

A more one-sided case may increase litigation expenditure if the following Plott CSF $\theta_W : \mathbb{R}^2_+ \to [0, 1]$ applies:

$$\theta_W(e_P, e_D) = \begin{cases} \eta \mu + (1 - \eta) \frac{e_P^{\gamma}}{e_P^{\gamma} + e_D^{\gamma}} & \text{if } e_P + e_D > 0, \\ \mu & \text{otherwise,} \end{cases}$$
(10)

with exogenous constants $0 < \gamma \le 1$, $0 < \eta < 1$. The weight η determines the relative influence of relative advantages μ and of litigation expenses on probabilities of success. An increase in η represents an increase in the relative weight that the judicial process gives to relative advantages, and a corresponding decrease in the relative weight given to expenses. If $\mu = 0.5$, then θ_W specializes to the one that Plott (1987) used to study the American and English rules.

Figure 1 depicts litigation expenditure \mathbb{E}^* as a function of Plaintiff's relative advantages μ in three special cases of the Monetary Litigation Game with half cost shifting, $\lambda = 0.5$. The blue dashed and red dotted curves in Fig. 1a capture the Tullock CSF (8) with exponent $\gamma = 1$, $\gamma = \frac{1}{2}$, respectively. In these cases, a more one-sided case reduces litigation expenditure. By comparison, the black solid curve in Fig. 1b adopts the Plott CSF (10) with exponent $\gamma = \frac{1}{2}$. In this case, litigation expenditure increases when the case becomes more one-sided.



Fig. 1 Litigation expenditure as a function of Plaintiff's relative advantages

4.3 Cost shifting and expenditure in litigated cases

Assumptions 1–8 do not guarantee that litigation expenditure is monotonic with the cost-shifting rule. Consider Fig. 2, in which the horizontal axis captures Plaintiff's relative advantages μ while the vertical axis $\frac{d\mathbb{E}^*}{d\lambda}$; this derivative indicates how litigation expenditure responds to infinitesimally more cost shifting. The purple solid curve represents the Tullock CSF (8) with exponent $\gamma = 1$ and cost-shifting rule $\lambda = 0.1$. In cases characterized by balanced relative advantages $\mu \in (\mu', \mu'')$, more cost shifting increases litigation expenditure, $\frac{d\mathbb{E}^*}{d\lambda} > 0$. In cases characterized by extreme relative advantages $\mu < \mu'$ or $\mu > \mu''$, more cost shifting *decreases* litigation expenditure, $\frac{d\mathbb{E}^*}{d\lambda} < 0.^{16}$

Motivated by Fig. 2, Corollary 4 will establish how cost shifting affects litigation expenditure in balanced cases (in a precise sense described below). Define functions $\bar{\theta}$: $[0, 1] \rightarrow [2/3, 0.75]$ and σ : $[0, 1] \rightarrow (0, 0.5]$ pointwise by

$$\bar{\theta}(\lambda) = \frac{3-\lambda}{4-\lambda}, \qquad \sigma(\lambda) = \max\left\{\mu \in [0,1] \,|\, \theta^* \le \bar{\theta}(\lambda)\right\} - 0.5. \tag{11}$$

Describing the radius of a close ball centering 0.5, function $\sigma(\cdot)$ first chooses the maximum relative-advantages parameter μ that induces an equilibrium probability θ^* no greater than $\bar{\theta}(\lambda)$, and then subtracts 0.5 from that μ .¹⁷

Corollary 4 Consider the equilibrium of the Monetary Litigation Game. Suppose the litigants' relative advantages are sufficiently balanced in the sense of $0.5 - \sigma(\lambda) \le \mu \le 0.5 + \sigma(\lambda)$. Then marginally more cost shifting increases litigation expenditure. Formally, $0.5 - \sigma(\lambda) \le \mu \le 0.5 + \sigma(\lambda)$ implies $\frac{d\mathbb{E}}{d\lambda} > 0$.

¹⁶ The existing litigation models do not obtain this result because they typically only consider $\lambda = 0$ and $\lambda = 1$ (see Sect. 1), or assume $\frac{de_D}{de_P} = 0$ (for example, Fenn et al. 2017 at pp. 147–148).

¹⁷ Lemma 5 in Appendix A confirms the existence of $\sigma(\cdot)$ and its range.



Fig. 2 How cost shifting affects litigation expenditure, with the cost-shifting rule $\lambda = 0.1$ as the baseline

Corollary 4 proves that if the litigants' relative advantages are sufficiently balanced (in the precise sense described by $\sigma(\cdot)$), then more cost shifting increases litigation expenditure in equilibrium.¹⁸ Intuitively, more cost shifting increases litigation expenditure if both litigants spend more, or if one litigant's additional spending is not offset by a more rapid reduction in the other litigant's spending. More cost shifting reduces a litigant's expected marginal costs by allowing a greater recovery of her costs if she wins. By increasing the recoverable-costs part of the "prize", more cost shifting also widens the difference in monetary outcome between wining and losing. A litigant must have very poor prospects to reduce expenses — which further harms her probability of success — in order to save costs. In cases characterized by sufficiently balanced relative advantages in the precise sense, no litigant has very poor prospects. Hence, in these cases, more cost shifting incentivizes the litigants collectively to spend more.

Remark 2 Introducing participation constraints into the Emotional Litigation Game does not resolve ambiguities regarding the relationship between cost shifting and litigation expenditure. Following Farmer and Pecorino (1999) (at p. 276) and Carbonara et al. (2015) (at pp. 120, 125), suppose Plaintiff obtains zero payoff upon not filing suit, her participation constraint is

$$\widehat{u}_P(e_P^*, e_D^*) \ge 0, \tag{12}$$

which captures the intuition that she would proceed to litigation only if she would do worse by not filing her case. Similarly, suppose Defendant can pay the judgment sum 1 instead of defending suit, her participation constraint is

$$\widehat{u}_D(e_P^*, e_D^*) \ge -1,\tag{13}$$

¹⁸ If participation constraints (12), 13) described in Remark 2 are introduced, then Corollary 4 assumes that these constraints are satisfied and non-binding. Otherwise an increase in cost shifting may lead to violation of a participation constraint, which means a litigant would be better off not filing or defending suit. In such a scenario, the increase in cost shifting reduces litigation expenditure to 0.

which captures the intuition that she would proceed to litigation only if she would do worse by not defending suit.

Assuming no emotions ($\xi = v = 0$) and specifying the Tullock CSF θ_T given by (8), Farmer and Pecorino (1999) and Carbonara et al. (2015) revealed that cases characterized by extreme relative advantages (μ) often do not satisfy the participation constraints. Among the findings of Farmer and Pecorino (at pp. 279–280) was that for some values of γ (the exponent in θ_T), under the American rule ($\lambda = 0$), Plaintiff would file suit only if μ is sufficiently large, and Defendant would defend suit only if μ is sufficiently small. They (at p. 284) obtained similar outcomes under the English rule ($\lambda = 1$) for some other values of γ . Carbonara et al. (2015) discovered a similar result under intermediate cost-shifting rules that limit the quantity of costs recoverable. They (at pp. 132–133) established that as the limiting quantity increases, fewer cases characterized by an extreme μ would proceed to litigation, and whether litigation would eventually cease depends on γ . Farmer and Pecorino (1999) (at pp. 273–274) did not consider settlement. Carbonara et al. (2015) (at p. 120, footnote 16) captured "settlement" in the sense of the defendant deciding to pay the judgment sum.

Under the present proportionate formulation of cost-shifting rules (which is more general than the quantity formulation, see footnote 9), imposition of participation constraints does not "rule out" imbalanced cases in the sense described by Corollary 4. Thus imposition of participation constraints does not imply that more cost shifting will necessarily increase litigation expenditure. To see this, consider a numerical example of the Monetary Litigation Game that adopts the Tullock CSF (8) with exponent $\gamma = 1$ and the American rule ($\lambda = 0$). Suppose the relative-advantages parameter is $\mu = 0.95$, which falls outside the scope of Corollary 4. A calculation exercise using Lemma 1, Proposition 1 and Appendix B shows that Plaintiff's and Defendant's equilibrium payoffs are exactly 0.9025, -0.9975, respectively; their participation constraints (12), (13) are satisfied and non-binding. The equilibrium litigation expenditure is 0.0950. Now suppose the cost-shifting rule increases to $\lambda = 0.01$. Then Plaintiff's and Defendant's equilibrium payoffs become 0.9032, -0.9980 to the fourth decimal place, respectively; their participation constraints are still satisfied and non-binding. Litigation expenditure now becomes 0.0948 to the fourth decimal place, which is smaller than that under the initial American rule. This example shows that in extreme cases, even if participation constraints are imposed, more cost shifting may still reduce litigation expenditure.

To provide intuition, Fig. 3a depicts the litigants' best reply functions in the numerical example involving the Tullock CSF (8) with $\gamma = 1$ and $\mu = 0.95$, in a neighbourhood of the equilibrium. Figure 3a shows the $(0.0470, 0.0478) \times (0.0470, 0.0478)$ space with Plaintiff's expenses e_P on the horizontal axis and Defendant's expenses e_D on the vertical axis. The black solid curve depicts Plaintiff's best reply as a function of e_D under the initial cost-shifting rule $\lambda_1 = 0$, while the green solid curve depicts Defendant's best reply as a function of e_P under $\lambda_1 = 0$. Their intersection at the exact point (0.0475, 0.0475) is the equilibrium under $\lambda_1 = 0$. Now suppose the extent of cost shifting increases to $\lambda_2 = 0.01$. The blue dotted curve and the red dashed curve depict Plaintiff's and Defendant's best reply functions under $\lambda_2 = 0.01$, respectively. The new equilibrium is (0.0476, 0.0472) to the fourth decimal place. The direct and



Fig. 3 Best reply functions given extreme relative advantages $\mu = 0.95$, in a neighbourhood of the equilibrium

indirect effects leading to this new equilibrium are

$$\frac{de_P^*}{d\lambda} = \underbrace{\frac{\partial e_P^*}{\partial \lambda}}_{\text{direct effect}} + \underbrace{\frac{de_P^*}{de_D}\frac{de_D^*}{d\lambda}}_{\text{indirect effect}} > 0 \quad \frac{de_D^*}{d\lambda} = \underbrace{\frac{\partial e_D^*}{\partial \lambda}}_{\text{direct effect}} + \underbrace{\frac{de_D^*}{de_D}\frac{de_P^*}{d\lambda}}_{\text{indirect effect}} < 0.$$

Given a fixed level of expenses by Defendant (respectively, Plaintiff), an increase in cost shifting from $\lambda_1 = 0$ to $\lambda_2 = 0.01$ has the direct effect of incentivising Plaintiff (Defendant) to spend more; formally, $\frac{\partial e_P^*}{\partial \lambda} > 0$ $\left(\frac{\partial e_D^*}{\partial \lambda} > 0\right)$. In Fig. 3a, such direct effect is depicted by an outward shift in Plaintiff's (Defendant's) best reply function. However, when Plaintiff has extremely strong relative advantages $\mu = 0.95$ and the CSF takes the Tullock form (8), Plaintiff's best reply function increases with e_D while Defendant's best reply function decreases with e_P near the equilibrium. Hence the increase in cost shifting from $\lambda_1 = 0$ to $\lambda_2 = 0.01$ also has the indirect effect of motivating Plaintiff to change her expenses in the same direction as changes in e_D ; formally, $\frac{de_p^*}{de_D} > 0$. The increase in cost shifting has the further indirect effect of motivating Defendant to change her expenses in the opposite direction as changes in e_P ; formally, $\frac{de_D^*}{de_P} < 0$. In fact, the rate of decrease in Defendant's best reply is much greater than the rate of increase in Plaintiff's best reply; formally, $\left|\frac{de_D^*}{de_P}\right| > \frac{de_P^*}{de_D}$. Overall, the indirect effect in terms of the sharp reduction in Defendant's best reply dominates the other indirect and direct effects. A smaller total litigation expenditure thus follows. As Fig. 3a shows, the increase from $\lambda_1 = 0$ to $\lambda_2 = 0.01$ reduces Defendant's expenses more than it increases Plaintiff's expenses in equilibrium.

To facilitate a comparison, Fig. 3b depicts the litigants' best reply functions given the Plott CSF (10) with $\gamma = 1$, $\eta = 0.5$ and $\mu = 0.95$, in a neighbourhood of the equilibrium. In the (0.1244, 0.1276) × (0.1244, 0.1276) space, the black solid curve and the green solid curve respectively depict Plaintiff's and Defendant's best reply functions under $\lambda_1 = 0$. Their intersection at the exact point (0.1250, 0.1250) is the equilibrium under $\lambda_1 = 0$, which amounts to 0.2500 in litigation expenditure. When the extent of cost shifting increases to $\lambda_2 = 0.01$, Plaintiff's and Defendant's best reply functions become the blue dotted curve and the red dashed curve, respectively. The new equilibrium is (0.1262, 0.1257) to the fourth decimal place. The litigation expenditure under $\lambda_2 = 0.01$ is 0.2519 to the fourth decimal place, which is greater than that under $\lambda_1 = 0$. Intuitively, more cost shifting has the direct effect of motivating a litigant to spend more in response to a fixed level of expenses by her adversary. Given the properties of the Plott CSF, the rate of change in each litigant's best reply function is almost zero near the equilibrium. In this case, each litigant's direct effect dominates the indirect effect. Hence more cost shifting leads to a greater total litigation expenditure under the Plott CSF.

A comparison of Fig. 3a, b shows that given an extreme $\mu = 0.95$, an increase in cost shifting from $\lambda_1 = 0$ to $\lambda_2 = 0.01$ leads to a greater litigation expenditure under the Plott CSF (10) but a smaller litigation expenditure under the Tullock CSF (8). This comparison illustrates that in extreme cases falling outside the scope of Corollary 4, whether more cost shifting increases or deceases litigation expenditure depends on the CSF.

4.4 Cost shifting and accuracy in adjudication

This section explores how changes in the extent of cost shifting affect the relationship between the litigants' relative advantages and their equilibrium probabilities of success. Define distortion $\Delta : (0, 1) \times [0, 1] \rightarrow \mathbb{R}$ by

$$\Delta(\mu, \lambda) = |\theta^* - \mu|, \tag{14}$$

where the equilibrium probability θ^* is a function of relative advantages μ and the costshifting rule λ . Intuitively, distortion measures the extent to which litigation expenses drive equilibrium probabilities of success away from the litigants' relative advantages. A large (respectively, small) distortion means that, compared to relative advantages, litigation expenses have a significant (insignificant) influence on equilibrium probabilities of success.

We label Δ as "distortion" without positive or negative connotation. Whether distortion is normatively desirable depends on interpretation of the exogenous and endogenous variables. For example, suppose the dispute between Plaintiff and Defendant concerns an injury suffered by Plaintiff which arose from both litigants' failure to take optimal care, and that a comparative negligence standard holds both of them liable for the cost of the injury in proportion to their respective departures from the levels of care required (Haddock and Curran 1985 at p. 50; Shavell 1987 at p. 15). Suppose further that we follow the norm in the litigation literature to interpret the relative-advantages parameter μ as the inherent merits of Plaintiff's case (for example, Katz 1988 at p. 129; Farmer and Pecorino 1999 at p. 272; Hirshleifer and Osborne 2001 at pp. 185–186; Carbonara et al. 2015 at p. 118). To capture this scenario, the sum in dispute 1 is interpreted as the normalized cost of the injury to Plaintiff, and



Fig. 4 Plaintiff's equilibrium probabilities of success as functions of her relative advantages

Defendant's optimal fractional share of that cost should be μ . Given the assumption that both litigants are risk-neutral, θ captures Defendant's fractional share of the cost 1 as a result of litigation. In this scenario, a non-zero distortion Δ is undesirable because it indicates a departure from the requirement of an optimal comparative negligence standard. Such distortion may affect injurers' incentives to take precaution, especially when they are informed (for example, Kaplow and Shavell 1996). Such distortion also may affect the desirability of the comparative negligence standard itself (for example, Cooter and Ulen 1986; Bar-Gill and Ben-Shahar 2003; Dari-Mattiacci and Hendriks 2013).

Alternatively, suppose the dispute between Plaintiff and Defendant concerns an injury suffered by Plaintiff, which was solely caused by Defendant's failure to take optimal care according to a negligence standard (Brown 1973 at p. 328; Shavell 1987 at p. 8). However, imperfection, bias or prejudice in the judicial system allows Defendant to escape liability with some positive probability (see footnote 11; Creswell and Johnston 1986 at pp. 284–285; Shavell 1987 at pp. 25, 79). To capture this scenario, interpret the sum in dispute 1 as the normalized cost of the injury to Plaintiff, and $1 - \mu$ as Defendant's probability of escaping liability for that cost due to judicial imperfection, bias or prejudice. Defendant's probability of escaping liability after accounting for the influence of litigation expenses is $1 - \theta$. In this scenario, a non-zero distortion Δ is desirable if it indicates $\theta^* > \mu$ in equilibrium, because Defendant's probability of escaping liability is reduced as a result of litigation expenses; formally, $1 - \theta^* < 1 - \mu$. However, a non-zero Δ is undesirable if it indicates $\theta^* > \mu$. Hence whether distortion is desirable is ambiguous in this alternative scenario.

Specification of the CSF affects the relationship between cost shifting and distortion. Consider two special cases of the Monetary Litigation Game depicted in Fig. 4. Based on the Tullock CSF (8) with exponent $\gamma = 1$, Fig. 4a plots the relationship between Plaintiff's equilibrium probability of success θ^* and her relative advantages μ under two cost-shifting rules $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$. The blue solid curve (respectively, green dashed curve) depicts the case of $\lambda_1 = 0.5$ ($\lambda_2 = 0.8$). For all $\mu \neq 0.5$, the value of θ^* on the green dashed curve is further away from μ compared to that on the blue solid curve. Now consider Fig. 4b, which is based on the Plott CSF (10) with exponent $\gamma = 1$. Again, the blue solid curve (respectively, green dashed curve) depicts the case of $\lambda_1 = 0.5$ ($\lambda_2 = 0.8$). For all $\mu \neq 0.5$, the value of θ^* on the green dashed curve is closer to μ compared to that on the blue solid curve. Thus more cost shifting generally increases distortion in the Tullock case but reduces distortion in the Plott case.

We now propose a new assumption to guarantee that more cost shifting generally increases distortion.

Assumption 9 If litigation expenses are positive and equal, then Plaintiff's probability of success equals her relative advantages. Formally, $e_P = e_D > 0$ implies $\theta = \mu$.

Assumption 9 imposes a condition in respect of all positive expenses pairs $(e_P, e_D) \in \mathbb{R}^2_{++}$, not just the equilibrium pair (e_P^*, e_D^*) . Intuitively, Assumption 9 requires probabilities of success to accurately reflect relative advantages if litigation expenses are equal. The Tullock CSF (8) satisfies Assumption 9, but the Plott CSF (10) does not.

Adding Assumption 9, Proposition 4 proves that the relatively more advantaged litigant has an equilibrium probability of success that is no smaller than her relative advantages. Let Θ_9 denote the set of CSFs $\theta : \mathbb{R}^2_+ \to [0, 1]$ that are continuous in $\mathbb{R}^2_+ \setminus \{(0, 0)\}$, twice continuously differentiable in \mathbb{R}^2_{++} , and satisfy Assumption 9.

Proposition 4 Consider the Monetary Litigation Game. Suppose there is no distortion when litigation expenses are equal, and one litigant is relatively more advantaged. Then this litigant's equilibrium probability of success is no smaller than her relative advantages. Her equilibrium probability of success is greater than her relative advantages if the cost-shifting rule allows positive recovery. Formally, $\theta \in \Theta(\{0\}, \{\lambda\}) \cap \Theta_9$ and $\mu > 0.5$ (respectively, $\mu < 0.5$) imply $\theta^* \ge \mu$ ($\theta^* \le \mu$), holding strictly if $\lambda > 0$.

Corollary 5 Consider the Monetary Litigation Game. Suppose there is no distortion when litigation expenses are equal, and one litigant is relatively more advantaged. Then marginally more cost shifting increases distortion in equilibrium. Formally, $\theta \in \Theta(\{0\}, \{\lambda\}) \cap \Theta_9$ and $\mu \neq 0.5$ imply $\frac{d\Delta}{d\lambda} > 0$.

Applying Proposition 4, Corollary 5 proves that the addition of Assumption 9 is sufficient for ensuring that more cost shifting generally increases distortion in equilibrium.¹⁹ Intuitively, Assumption 9 guarantees that in any unequal-advantages case and under any cost-shifting rule, the equilibrium probability of success of the more advantaged litigant is no smaller than her relative advantages. Then more cost shifting increases relative expenses in her favor (Corollary 2), which pushes her equilibrium probability of success further above her relative advantages.

¹⁹ If participation constraints (12), 13) described in Remark 2 are introduced, then Corollary 5 assumes that these constraints are satisfied and non-binding. Otherwise an increase in cost shifting may lead to Plaintiff (respectively, Defendant) not filing suit (defending suit), which implies that distortion given by (14) equals μ (respectively, $1 - \mu$).

	Tullock θ_T	Plott θ_W	A1–A8	A1-A8 and A9	A1–A8 and $ \mu \le 0.5 + \sigma$	
$\frac{d\mathbb{E}^*}{d\lambda}$	Depends	> 0	Depends	Depends	> 0	
$\frac{d\Delta}{d\lambda}$	> 0	< 0	Depends	> 0	Depends	

Table 1 Effects of cost shifting (λ) on litigation expenditure (\mathbb{E}^* given by (9)) and distortion (Δ given by (14))

Summarizing the findings of Sects. 4.3 and 4.4, Table 1 describes how changes in cost shifting affect litigation expenditure and distortion in the Monetary Litigation Game.

5 Variations in emotions

This section ascertains how variations in emotions affect equilibrium properties in the Emotional Litigation Game.

Assuming the CSF satisfies Assumption 9 in addition to Assumptions 1–8, Corollary 6 proves that more negative relational emotions amplifies the distortionary effect of positive cost shifting in equilibrium. This result follows from Proposition 2 and the finding that more cost shifting increases distortion in equilibrium (Corollary 5).

Corollary 6 Consider the equilibrium of the Emotional Litigation Game. Suppose there is no distortion when litigation expenses are equal, one litigant is relatively more advantaged, and the cost-shifting rule allows positive recovery. Then as relational emotions become marginally more negative, distortion increases. Formally, $\theta \in \Theta(\{0\}, \{\widehat{\lambda}\}) \cap \Theta_9$, $\mu \neq 0.5$ and $\widehat{\lambda} > 0$ imply $\frac{d\Delta}{d\varepsilon} < 0$.

Without imposing Assumption 9, Corollary 7 considers how changes in relational emotions ξ affect various equilibrium variables. More negative relational emotions not only directly increase the emotional reward of winning, but also indirectly amplify the effects of cost shifting (Proposition 2). Intuitively, when a litigant has more negative relational emotions, she derives more value from harming her adversary both through winning the lawsuit and through shifting costs onto her adversary. Any asymmetric effects of more negative relational emotions ξ only arise from its interaction with the cost-shifting rule.

Corollary 7 Consider the equilibrium of the Emotional Litigation Game.

- 1. Suppose the litigants' relative advantages are equal or the American rule applies. Then the litigants' relative expenses and probabilities of success do not change as relational emotions ξ change. Formally, $\mu = 0.5$ or $\hat{\lambda} = 0$ implies $\frac{ds^*}{d\xi} = 0$, $\frac{d\theta^*}{d\xi} = 0$.
- 2. Suppose the American rule applies. Then as relational emotion become marginally more negative, litigation expenditure increases. Formally, $\hat{\lambda} = 0$ implies $\frac{d\mathbb{E}^*}{d\xi} < 0$.
- 3. Suppose the cost-shifting rule allows positive recovery. Suppose further that Plaintiff (respectively, Defendant) is relatively more advantaged. Then marginally more

negative relational emotions (ξ decreases) increase her expenses relative to Defendant's (Plaintiff's) and her probability of success. Formally, λ > 0 and μ > 0.5 (μ < 0.5) imply ds^{*}/dξ > 0, dθ^{*}/dξ < 0 (ds^{*}/dξ > 0).
4. Suppose the litigants' relative advantages are sufficiently balanced. Then as relative advantages are sufficiently balanced.

4. Suppose the litigants' relative advantages are sufficiently balanced. Then as relational emotion become marginally more negative, litigation expenditure increases. Formally, $0.5 - \sigma(\lambda) \le \mu \le 0.5 + \sigma(\lambda)$ implies $\frac{d\mathbb{E}^*}{d\xi} < 0$.

Part 1 of Corollary 7 establishes that if the litigants have equal relative advantages $(\mu = 0.5)$, then more negative relational emotions ξ do not vary the equilibrium relative expenses and probabilities of success. Moreover, in cases where the American rule applies $(\hat{\lambda} = 0)$, there is "nothing" for ξ to amplify. Hence part 1 also proves that in these cases, a more negative ξ does not change the equilibrium relative expenses and probabilities of success. Part 2 establishes that under the American rule, more negative relational emotions ξ leads to a greater total litigation expenditure in equilibrium. In addition, part 3 reveals that in cases involving unequal relative advantages and positive cost shifting ($\mu \neq 0.5$ and $\hat{\lambda} > 0$), a more negative ξ amplifies the asymmetric effects of cost shifting, which results in greater relative expenses and probability of success for the relatively more advantaged litigant. Finally, part 4 proves that a more negative ξ increases litigation expenditure in cases involving sufficiently balanced relative advantages. A comparison of Corollary 7 with Corollaries 2, 4 shows that more negative relational emotions and more cost shifting have similar effects on equilibrium variables.

Corollary 8 ascertains how changes in the joy of winning v affect various equilibrium variables, including individual monetary payoffs in equilibrium. Let u_p^* and u_D^* respectively denote Plaintiff's and Defendant's equilibrium monetary payoffs (see equations (1), (2)).

Corollary 8 Consider the equilibrium of the Emotional Litigation Game.

- 1. Variations in the joy of winning υ do not affect the litigants' relative expenses or probabilities of success. Formally, $\frac{ds^*}{d\upsilon} = 0$, $\frac{d\theta^*}{d\upsilon} = 0$.
- 2. Variations in the joy of winning do not affect distortion. Formally, $\frac{d\Delta}{dv} = 0$.
- 3. A greater joy of winning reduces each litigant's monetary payoff. Formally, $\frac{du_p^*}{dv} < 0$, $\frac{du_p^*}{dv} < 0$.
- 4. A greater joy of winning leads to a greater litigation expenditure. Formally, $\frac{d\mathbb{E}^*}{dv} > 0$.

Unlike relational emotions, the joy of winning v does not interact with the costshifting rule and only affects the litigants in a symmetric manner (Proposition 2). To the same extent for both litigants, a greater v increases the marginal benefits of spending to win. Part 1 of Corollary 8 thus establishes that equilibrium relative expenses and probabilities of success remain constant when v changes. Moreover, given distortion (14) depends on relative expenses rather than their magnitude, part 2 shows that variations in v do not affect distortion in equilibrium. In addition, given the absence of interaction between v and the cost-shifting rule, no variable in this model offsets the heightened incentive to spend arising from a greater v. Hence parts 3 and 4 respectively prove that a greater v leads to a lower individual monetary payoff and a greater total litigation expenditure in equilibrium. These last two results hold even in cases involving extreme relative advantages, which fall outside the scope of Corollary 4 and part 4 of Corollary 7. Baumann and Friehe (2012) (at pp. 196, 203– 204) obtained similar results in a Tullock model with no cost shifting, and the present Corollary 8 confirms these results when the CSF is generally-formulated and arbitrary cost shifting is introduced.

6 Discussion

This section elicits several normative implications of the Emotional Litigation Game to the extent that litigation expenses are the only endogenous variables. It concludes with some suggestions for future research.

Our main result concerns the strategic interaction of cost-shifting rules and negative relational emotions-meaning a litigant derives value from harming her adversary. Proposition 2 in Sect. 3 reveals that negative relational emotions *amplify* the effects of cost shifting. This result implies that models based on purely self-interested litigants underestimate the full effects of cost shifting. The presence of preferences to harm an adversary is intuitively sound in many litigated cases, empirically supported in divorce cases (Farmer and Tiefenthaler 2001), and frequently observed in contest experiments (see Sect. 1). Thus a nominally low-powered cost-shifting rule can have the practical implications of a higher-powered rule. If a lawmaker (or a judge, when she has discretion over cost shifting) aims to effectuate a particular extent of cost shifting, then in the presence of strong negative relational emotions she should apply a nominally weaker cost-shifting rule. Mathematically, in cases involving negative relational emotions $\xi < 0$, to effect uate the effects of a cost-shifting rule characterized by the loser bearing $0 < \lambda \leq 1$ proportion of the winner's costs requires application of a weaker rule characterized by $\hat{\lambda} = \lambda/(1-\xi)$. However, if the goal is to minimize the impact of negative relational emotions, then the American rule-that each litigant bears her own costs-should be applied. Negative relational emotions amplify the effects of cost shifting, but there is "nothing" to amplify if there is no cost shifting; formally, $\lambda = 0$ implies $\hat{\lambda} = 0$ regardless of ξ .

In reality, while many common law jurisdictions apply high-powered cost-shifting rules by default, judges often exercise their discretion to effectuate low-powered cost shifting in cases involving emotionally-charged litigants with intertwined and conflicted interests. A prominent example concerns inheritance disputes (Vines 2011 at pp. 11–13). Our analysis offers a behavioral-economic foundation for that judicial practice. Given emotions can be hard to observe, measure or verify in court, we suggest that low-powered cost-shifting rules should apply by default, rather than by discretion, in those classes of cases that typically exhibit strong negative relational emotions. For instance, low-powered cost-shifting rules should apply by default in inheritance and divorce cases, where anger, feelings of injustice or other negative visceral factors can motivate litigants to inflict harm upon each other (see Farmer and Tiefenthaler 2001; Loewenstein 2000 at pp. 429–430).

Moreover, our analysis suggests that the choice of cost-shifting rules should not depend on whether litigants are driven by a joy of winning, which they derive from winning the lawsuit rather than harming their adversaries. A real-life example concerns disputes over properties that have a high non-monetary value.²⁰ Unlike relational emotions, such outcome-dependent value does not modify the effects of cost shifting (see Proposition 2).

Future research may modify the Emotional Litigation Game to study settlement negotiation. Existing settlement models typically assume that litigants have purely monetary preferences and that once a case proceeds to trial, litigation expenses do not vary with the extent of cost shifting (see footnote 2). The small literature that accounts for emotions do not consider intermediate cost shifting (see footnote 6). The Emotional Litigation Game reveals the combined effects of cost-shifting rules and emotions on endogenous litigation expenses (see, in particular, Corollaries 4, 7). Given litigation expenses at trial are also the monetary surplus of settlement, future research may explore how the present results regarding the interplay of emotions and cost-shifting rules may affect incentives to settle.

Future research may also apply the present model to study research and development, cross-shareholding, or other contests with spillovers (see Sect. 1). Our equilibrium existence and uniqueness result permits a large class of CSFs, two asymmetric players with monetary or emotional preferences, and proportionate spillovers. This result may be developed further to study contests other than litigation.

A Appendix: Proofs

This appendix contains all proofs. Table 2 lists the proofs in order of appearance and identifies the previous results used in each proof. Lemma 2 will be used to prove other Lemmas, Propositions and Corollaries.

Lemma 2 In the subdomain \mathbb{R}^2_{++} , the CSF $\theta(\cdot)$ satisfies the following properties: 1. $\mu > 0.5$ (respectively, = 0.5, < 0.5) and $e_P = e_D$ imply $\theta > 0.5$ (= 0.5, < 0.5). 2. $\frac{\partial}{\partial e_D}(1-\theta) > 0$, $\frac{\partial^2 \theta}{\partial e_P^2} \le 0$, and $\frac{\partial^2}{\partial e_D^2}(1-\theta) \le 0$. 3.

$$\frac{\frac{\partial^2}{\partial e_D^2} \left(\frac{1-\theta}{1-\widehat{\lambda}(1-\widehat{\xi})(1-\theta)} \right)}{\frac{\partial}{\partial e_D} \left(\frac{1-\theta}{1-\widehat{\lambda}(1-\widehat{\xi})(1-\theta)} \right)} < 0.$$

 $4. \ \widehat{\lambda}(1-\xi) = 1 \Rightarrow \lim_{s \to +\infty} \theta > 0.$ $5. \ \frac{\partial s}{\partial e_P} = -\frac{s}{e_P}, \ \frac{\partial s}{\partial e_D} = \frac{s}{e_D}, \ \frac{\partial \theta}{\partial e_P} = -\frac{s\theta_s}{e_P}, \ \frac{\partial}{\partial e_D}(1-\theta) = -\frac{s\theta_s}{e_D}, \ \frac{\partial^2 \theta}{\partial e_P^2} = \frac{s^2\theta_{ss}}{e_P^2} + \frac{2s\theta_s}{e_P^2}, \ \frac{\partial^2 \theta}{\partial e_P^2}(1-\theta) = -\frac{s^2\theta_{ss}}{e_D^2}, \ \frac{\partial^2 \theta}{\partial e_P \partial e_D} = -\frac{s(\theta_s + s\theta_{ss})}{e_P e_D}.$

²⁰ For example, in US common law jurisdictions "land has long been regarded as unique and impossible of duplication by the use of any amount of money" (American Law Institute, *Restatement (Second) of Contracts* (1981) §360, comment e). Restatements authoritatively state prevailing US law.

Table 2 List of proofs and previc	ous results used in each proof		
Proof (sorted \)	Previous results used in proof	Proof (sorted ↓)	Previous results used in proof
Lemma 2		Lemma 4	Lemmas 1, 2, Proposition 1
Lemma 3		Lemma 5	Proposition 1, Corollary 3
Lemma 1	Lemma 2	Lemma 6	Lemmas 1, 4, Corollary 2
Proposition 1	Lemmas 1, 3	Corollary 4	Lemmas 1, 2, 4, 6, Proposition 1, Corollaries 1, 2
Proposition 2	Lemma 1, Proposition 1	Proposition 4	Lemmas 1, 2, Proposition 1
Corollary 1	Proposition 1	Corollary 5	Corollaries 1, 2, Proposition 4
Proposition 3		Corollary 6	Proposition 2, Corollary 5
Corollary 2	Lemmas 1, 2, Proposition 1	Corollary 7	Propositions 1, 2, Corollaries 2, 4
Corollary 3	Lemma 1, Proposition 1	Corollary 8	Lemma 1, Propositions 1, 2

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 $\begin{aligned} 6. \ \theta_s &< 0, \ \theta_{ss} \geq 0. \\ 7. \ s\theta_{ss} &< -2\theta_s - 2\widehat{\lambda}(1-\xi)s\theta_s^2/[1-\widehat{\lambda}(1-\xi)\theta]. \\ 8. \ s\theta_{ss} &> 2\widehat{\lambda}(1-\xi)s\theta_s^2/[1-\widehat{\lambda}(1-\xi)(1-\theta)]. \\ 9. \ -\widehat{\lambda}(1-\xi)(2-\widehat{\lambda}(1-\xi))s\theta_s &< (1-\widehat{\lambda}(1-\xi)\theta)[1-\widehat{\lambda}(1-\xi)(1-\theta)]. \end{aligned}$

Proof of Lemma 2 Part 1: Let $e_P = e_D = e_1$ for an arbitrary $e_1 > 0$. Suppose $\mu = 0.5$. From Assumption 1, obtain $\theta(e_1, e_1; 0.5) = 1 - \theta(e_1, e_1; 1 - 0.5) = 1 - \theta(e_1, e_1; 0.5)$, which implies $\theta(e_1, e_1; \mu) = 0.5$. Assumption 3 implies the result for $\mu \neq 0.5$.

Parts 2–4: Fix μ and $e_P = e_1$ for some arbitrary $e_1 > 0$. Assumption 1 implies $1 - \theta(e_1, e_D, \mu) = \theta(e_D, e_1; 1 - \mu)$. Hence $\frac{\partial}{\partial e_D}(1 - \theta(e_1, e_D; \mu)) = \frac{\partial}{\partial e_D}\theta(e_D, e_1; 1 - \mu) > 0$, where the inequality follows from Assumption 4.

Similar steps using Assumptions 5, 7 and inequality (15) prove $\frac{\partial^2 \theta}{\partial e_P^2} \leq 0$, $\frac{\partial^2}{\partial e_D^2}(1 - \theta(e_P, e_D; \mu)) \leq 0$, parts 3, 4.

Part 5: The chain rule, Young's theorem and some algebra will give these results.

Part 6: The chain rule and $\partial \theta / \partial e_P > 0$, $s/e_P > 0$ imply $\theta_s < 0$. Parts 2, 5 imply $\theta_{ss} \ge 0$.

Part 7: Some algebra will reveal that inequality (7) holds if and only if

$$(1 - \widehat{\lambda}(1 - \xi)\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\widehat{\lambda}(1 - \xi)\left(\frac{\partial\theta}{\partial e_P}\right)^2 < 0.$$
(15)

Then use parts 5, 6 to obtain

$$\begin{split} &(1-\widehat{\lambda}(1-\xi)\theta)s\theta_{ss}+2\theta_s(1-\widehat{\lambda}(1-\xi)\theta)+2\widehat{\lambda}(1-\xi)s\theta_s^2<0\\ &\Leftrightarrow s\theta_{ss}<-2\theta_s-\frac{2\widehat{\lambda}(1-\xi)s\theta_s^2}{1-\widehat{\lambda}(1-\xi)\theta}. \end{split}$$

Part 8: Some algebra will reveal that part 3 holds if and only if

$$-[1-\widehat{\lambda}(1-\xi)(1-\theta)]\frac{\partial^2\theta}{\partial e_D^2} + 2\widehat{\lambda}(1-\xi)\left(\frac{\partial\theta}{\partial e_D}\right)^2 < 0.$$

Then use part 5 to obtain

$$-[1-\widehat{\lambda}(1-\xi)(1-\theta)]s\theta_{ss}+2\widehat{\lambda}(1-\xi)s\theta_{s}^{2}<0 \Leftrightarrow \frac{2\widehat{\lambda}(1-\xi)s\theta_{s}^{2}}{1-\widehat{\lambda}(1-\xi)(1-\theta)}< s\theta_{ss}.$$

Part 9: Using $\theta_s < 0$, some algebra will derive the result from parts 7 and 8.

Lemma 3 is a technical lemma which allows any equilibrium of the Emotional Litigation Game to be characterized by a system of first order conditions (FOCs).

Lemma 3 Each litigant's emotional payoff function is strictly quasiconcave in her own expenses.

Proof of Lemma 3 This proof establishes the result for Plaintiff by taking the following steps: (i) establish that if Plaintiff's FOC holds at a pair of expenses, then her SOC is negative at that pair; (ii) apply a theorem from Diewert et al. (1981) to the result of step (i). Defendant's result follows symmetric steps.

Step (*i*): Take the partial derivatives of Plaintiff's emotional payoff function (4) with respect to her expenses e_P :

$$\frac{\partial \widehat{u}_P}{\partial e_P} = \frac{\partial \theta}{\partial e_P} (1 - \xi) [1 + \upsilon + \widehat{\lambda} e_P + \widehat{\lambda} e_D] - [1 - \widehat{\lambda} (1 - \xi) \theta]$$
(16)

$$\frac{\partial^2 \widehat{u}_P}{\partial e_P^2} = \frac{\partial^2 \theta}{\partial e_P^2} (1 - \xi) [1 + \upsilon + \widehat{\lambda} e_P + \widehat{\lambda} e_D] + 2\widehat{\lambda} (1 - \xi) \frac{\partial \theta}{\partial e_P}.$$
 (17)

Suppose Plaintiff's FOC holds. Then substituting (16) into (17) obtains

$$\begin{aligned} \frac{\partial^{2} \widehat{u}_{P}}{\partial e_{P}^{2}} &= \frac{\partial^{2} \theta}{\partial e_{P}^{2}} \bigg[\frac{[1 - \widehat{\lambda}(1 - \xi)\theta]}{\partial \theta / \partial e_{P}} \bigg] + 2\widehat{\lambda}(1 - \xi) \frac{\partial \theta}{\partial e_{P}} \\ &= [1 - \widehat{\lambda}(1 - \xi)\theta] \Bigg[\frac{[1 - \widehat{\lambda}(1 - \xi)\theta]\frac{\partial^{2} \theta}{\partial e_{P}^{2}} + 2\widehat{\lambda}(1 - \xi)\left(\frac{\partial \theta}{\partial e_{P}}\right)^{2}}{[1 - \widehat{\lambda}(1 - \xi)\theta]\frac{\partial \theta}{\partial e_{P}}} \Bigg] < 0 \end{aligned}$$

where the last inequality uses Assumptions 5, 7.

Step (ii). Corollary 9.3 of Diewert et al. (1981) holds that a continuously differentiable function f defined on an open S is strictly quasiconcave iff $y^0 \in S$, $w^T w = 1$ and $w^T \nabla f(y^0) w = 0$ implies $w^T \nabla^2 f(y^0) w < 0$; or $w^T \nabla^2 f(y^0) w = 0$ and $g(z) \equiv f(y^0 + zw)$ does not attain a local minimum at z = 0. We apply their result.

Fix $e_D = e_1$ for an arbitrary $e_1 > 0$. Consider Plaintiff's emotional payoff function \widehat{u}_P . Suppose $e_P > 0$, $w^T w = 1$ and $0 = w^T \nabla \widehat{u}_P(e_P, e_1) w = w^T \frac{\partial}{\partial e_P} \widehat{u}_P(e_P, e_1) w$. That $w^T w = 1$ implies $w \neq 0$. Hence $\frac{\partial}{\partial e_P} \widehat{u}_P(e_P, e_1) = 0$. Then step (*i*) proves $0 > \frac{\partial^2}{\partial e_P^2} \widehat{u}_P(e_P, e_1) = \nabla^2 \widehat{u}_P(e_P, e_1)$. That $w \neq 0$ implies $w^T \nabla^2 \widehat{u}_P(e_P, e_1) w < 0$. Then Corollary 9.3 of Diewert et al. (1981) implies that \widehat{u}_P is strictly quasiconcave in e_P .

Proof of Lemma 1 Define a function $h : \mathbb{R}_{++} \to \mathbb{R}$ by:

$$h(s) = 1 - \widehat{\lambda}(1 - \xi)\theta - s[1 - \widehat{\lambda}(1 - \xi)(1 - \theta)].$$
(18)

The first two steps of this proof establish the existence of some s^* such that $h(s^*) = 0$, and its value relative to 0.5, in the following two cases: (i) $\mu = 0.5$ or $\hat{\lambda} = 0$; and (ii) $\mu > 0.5$ and $\hat{\lambda} > 0$. (The case of $\mu < 0.5$ and $\hat{\lambda} > 0$ follows similar steps as (ii).) Step (iii) establishes uniqueness.

Step (i): Let $\mu = 0.5$. Part 1 of Lemma 2 implies that choosing $s^* = 1$ induces $\theta = 0.5 = 1 - \theta$. Then h(1) = 0.

Now suppose $\hat{\lambda} = 0$, which implies $\hat{\lambda}(1 - \xi) = 0$. Choosing $s^* = 1$ induces $h(s^*) = 0$.

Step (ii): Suppose $\mu > 0.5$ and $\widehat{\lambda} > 0$. Define a new function $h_1(s)$ by:

$$h_1(s) = \frac{h(s)}{s} = \frac{1 - \hat{\lambda}(1 - \xi)\theta}{s} - [1 - \hat{\lambda}(1 - \xi)(1 - \theta)].$$
(19)

Part 1 of Lemma 2 implies that s = 1 induces $\theta(1; \mu) > 0.5 > 1 - \theta(1; \mu)$. Some algebra obtains:

$$1 - \widehat{\lambda}(1 - \xi)\theta(1; \mu) < 1 - \widehat{\lambda}(1 - \xi)(1 - \theta(1; \mu)) \qquad \Leftrightarrow \qquad h_1(1) < 0.$$

Now, consider the limit of $h_1(s)$ as *s* approaches 0:

$$\lim_{s \to 0} h_1(s) = \lim_{s \to 0} \left(\frac{1 - \widehat{\lambda}(1 - \xi)\theta}{s} - [1 - \widehat{\lambda}(1 - \xi)(1 - \theta)] \right)$$
$$= \lim_{s \to 0} \left(\frac{1 - \widehat{\lambda}(1 - \xi)\theta}{s} \right) - 1 + \lim_{s \to 0} [\widehat{\lambda}(1 - \xi)(1 - \theta)].$$

Assumption 6 implies $\lim_{s\to 0} (\hat{\lambda}(1-\xi)\theta) < 1$. Then $\lim_{s\to 0} h_1(s) = +\infty$. This, $h_1(1) < 0$ and the intermediate value theorem imply there exists some $s^* \in (0, 1)$ satisfying $h_1(s^*) = 0$. The definition of h_1 in (19) implies $h(s^*) = 0$.

Step (iii): $h(\cdot)$ is continuously differentiable. Obtain $h'(s) = -[1 - \hat{\lambda}(1 - \xi)(1 - \theta)] - \hat{\lambda}(1 - \xi)(1 + s)\theta_s$. For any s > 0 satisfying h(s) = 0, some algebra obtains $s = [1 - \hat{\lambda}(1 - \xi)\theta]/[1 - \hat{\lambda}(1 - \xi)(1 - \theta)]$. Substitue into $h'(\cdot)$ gives

$$h'(s) = -\frac{(1-\widehat{\lambda}(1-\xi)\theta)[1-\widehat{\lambda}(1-\xi)(1-\theta)] + \widehat{\lambda}(1-\xi)(2-\widehat{\lambda}(1-\xi))s\theta_s}{s[1-\widehat{\lambda}(1-\xi)(1-\theta)]} < 0$$

where the last inequality uses part 9 of Lemma 2. Hence h'(s) < 0 whenever h(s) = 0.

Suppose, for a contradiction, there exist two different $s^{*'} > s^* > 0$ satisfying $h(s^{*'}) = h(s^*) = 0$, and $h(s') \neq 0$ for all $s' \in (s^*, s^{*'})$. Then $h'(s^*), h'(s^*) < 0$ imply $h(s^* + \epsilon) < 0$ and $h(s^{*'} - \epsilon) > 0$ for a small $\epsilon > 0$. The intermediate value theorem implies there exists some s'' > 0 satisfying $s^* < s'' < s^{*'}$ and h(s'') = 0, a contradiction. Thus there exists exactly one s^* satisfying $h(s^*) = 0$.

Proof of Proposition 1 By Theorem 8 of Diewert et al. (1981), any local maximizer of a strictly quasiconcave function is the unique global maximizer. Lemma 3 implies a litigant's FOC characterizes her best reply. A substitution exercise using equations (4), (5) reveals that $(e_P, e_D) \in \mathbb{R}^2_{++}$ constitutes a Nash equilibrium if and only if it satisfies

$$\begin{cases} 0 = \frac{\partial\theta}{\partial e_P} (1-\xi) [1+\upsilon+\widehat{\lambda}e_P+\widehat{\lambda}e_D] - [1-\widehat{\lambda}(1-\xi)\theta] \\ 0 = \frac{\partial(1-\theta)}{\partial e_D} (1-\xi) [1+\upsilon+\widehat{\lambda}e_P+\widehat{\lambda}e_D] - [1-\widehat{\lambda}(1-\xi)(1-\theta)]. \end{cases}$$
(20)

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This proof will first establish that (e_P^*, e_D^*) as described by this Proposition satisfies both FOCs in system (20). The proof will then prove the other direction and uniqueness.

Step (i): Let $s = s^*$ and use the expression for e_P^* to obtain

$$e_P^* = \frac{-(1+\nu)(1-\xi)s\theta_s}{s[1-\widehat{\lambda}(1-\xi)(1-\theta)] + \widehat{\lambda}(1-\xi)(1+s)s\theta_s} = \frac{-(1+\nu)(1-\xi)s\theta_s}{[1-\widehat{\lambda}(1-\xi)\theta] + \widehat{\lambda}(1-\xi)(1+s)s\theta_s}$$

where the last equality uses $1 - \hat{\lambda}(1-\xi)\theta = s[1-\hat{\lambda}(1-\xi)(1-\theta)]$ from Lemma 1. Then

$$\frac{-(1+\upsilon)(1-\xi)s\theta_s}{e_P^*} = [1-\widehat{\lambda}(1-\xi)\theta] + \widehat{\lambda}(1-\xi)(1+s)s\theta_s$$

$$\Leftrightarrow \ 1-\widehat{\lambda}(1-\xi)\theta = \frac{-(1-\xi)s\theta_s}{e_P^*} [1+\upsilon+\widehat{\lambda}(1+s)e_P^*]$$

$$= \frac{\partial\theta}{\partial e_P} (1-\xi) [1+\upsilon+\widehat{\lambda}e_P^*+\widehat{\lambda}e_D^*]$$

which proves that (e_P^*, e_D^*) satisfies Plaintiff's FOC.

Now consider the expression for e_D^*

$$e_D^* = se_P^* = \frac{-(1+\nu)(1-\xi)s^2\theta_s}{s[1-\hat{\lambda}(1-\xi)(1-\theta)] + \hat{\lambda}(1-\xi)(1+s)s\theta_s}$$

a rearrangement of which gives:

$$\frac{-(1+\upsilon)(1-\xi)s\theta_s}{e_D^*} = 1 - \widehat{\lambda}(1-\xi)(1-\theta) + \widehat{\lambda}(1-\xi)(1+s)\theta_s$$

$$\Leftrightarrow \ 1 - \widehat{\lambda}(1-\xi)(1-\theta) = \frac{-(1+\upsilon)(1-\xi)s\theta_s}{e_D^*} - \widehat{\lambda}(1-\xi)(1+s)\theta_s$$

$$= \frac{\partial(1-\theta)}{\partial e_D}(1-\xi) \Big[1+\upsilon + \widehat{\lambda}e_P^* + \widehat{\lambda}e_D^*\Big]$$

which proves (e_P^*, e_D^*) satisfies Defendant's FOC.

Step (ii): Let $(e'_P, e'_D) \in \mathbb{R}^2_{++}$ be a Nash equilibrium with positive expenses. Let $s' = e'_D/e'_P$. Then

$$e'_P = \frac{e'_P + e'_D}{1 + s'}$$
 $e'_D = \frac{s'(e'_P + e'_D)}{1 + s'}.$

Substitute these into the FOCs in system (20) to obtain

$$\frac{-(1+\upsilon)(1-\xi)s(1+s)\theta_s}{[1-\widehat{\lambda}(1-\xi)\theta] + \widehat{\lambda}(1-\xi)(1+s)s\theta_s}\bigg|_{s=s'}$$

= $e'_P + e'_D = \frac{-(1+\upsilon)(1-\xi)s(1+s)\theta_s}{s[1-\widehat{\lambda}(1-\xi)(1-\theta)] + \widehat{\lambda}(1-\xi)(1+s)s\theta_s}\bigg|_{s=s'}$

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where the first equality (respectively, second equality) is derived from Plaintiff's (Defendant's) FOC. Then some algebra using the equality of both sides will reveal that s = s' induces $1 - \hat{\lambda}(1 - \xi)\theta = s[1 - \hat{\lambda}(1 - \xi)(1 - \theta)]$. Hence the uniqueness limb of Lemma 1 implies $s' = s^*$.

Then obtain from the definition of e_P^* in Proposition 1

$$e'_{P} + e'_{D} = \frac{-(1+\upsilon)(1-\xi)s(1+s)\theta_{s}}{s[1-\widehat{\lambda}(1-\xi)(1-\theta)] + \widehat{\lambda}(1-\xi)(1+s)s\theta_{s}} \bigg|_{s=s^{*}=s'} = (1+s^{*})e_{P}^{*}$$

where $e'_P + e'_D = (1 + s')e'_P$, $s' = s^*$ imply $e'_P = e^*_P$. Similarly, $e'_P + e'_D = e'_D(1 + s')/s'$, $s' = s^*$ imply $e'_D = e^*_D$.

Proof of Proposition 2 An application of Lemma 1 and Proposition 1 gives the result. □

Proof of Corollary 1 Suppose θ satisfies Assumptions 1–8 given a pair ξ_0 , $\hat{\lambda}_0$. Consider an arbitrary pair ξ , $\hat{\lambda}$ that satisfies $\hat{\lambda}(1 - \xi) \leq \hat{\lambda}_0(1 - \xi_0)$. This proof will establish Assumptions 1–8 given ξ , $\hat{\lambda}$. Then applying Proposition 1 to ξ , $\hat{\lambda}$ gives the result.

Satisfaction of Assumptions 1–4, 8 does not depend on ξ , $\hat{\lambda}$. Assumption 5 is satisfied given $\hat{\lambda}(1-\xi) \leq \hat{\lambda}_0(1-\xi_0)$. Assumption 6 is satisfied if $\hat{\lambda}(1-\xi) = \hat{\lambda}_0(1-\xi_0) = 1$, or not applicable otherwise. It remains to prove Assumption 7.

Some algebra using the property that θ satisfies Assumption 7 under the pair ξ_0 , $\widehat{\lambda}_0$ reveals

$$0 > \frac{\frac{\partial^2}{\partial e_P^2} \left(\frac{\theta}{1-\widehat{\lambda}_0(1-\xi_0)\theta}\right)}{\frac{\partial}{\partial e_P} \left(\frac{\theta}{1-\widehat{\lambda}_0(1-\xi_0)\theta}\right)} \\ = \frac{\left(1-\widehat{\lambda}_0(1-\xi_0)\theta\right)\frac{\partial^2\theta}{\partial e_P^2} + 2\widehat{\lambda}_0(1-\xi_0)\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}} \\ \ge \frac{\left(1-\widehat{\lambda}(1-\xi)\theta\right)\frac{\partial^2\theta}{\partial e_P^2} + 2\widehat{\lambda}(1-\xi)\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}}$$

where the last weak inequality uses $\hat{\lambda}(1-\xi) \leq \hat{\lambda}_0(1-\xi_0), 1-\hat{\lambda}_0(1-\xi_0)\theta \leq 1-\hat{\lambda}(1-\xi)\theta$, and part 2 of Lemma 2. That θ satisfies Assumption 7 given $\xi, \hat{\lambda}$ follows from some algebra revealing

$$\frac{\frac{\partial^2}{\partial e_P^2} \left(\frac{\theta}{1-\widehat{\lambda}(1-\xi)\theta}\right)}{\frac{\partial}{\partial e_P} \left(\frac{\theta}{1-\widehat{\lambda}(1-\xi)\theta}\right)} = \frac{(1-\widehat{\lambda}(1-\xi)\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\widehat{\lambda}(1-\xi)\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}}.$$

The choice of ξ , $\hat{\lambda}$ was arbitrary.

Proof of Proposition 3 This proof will suppose Defendant chooses $e_D = 0$, and show that (i) $e_P = 0$ is not Plaintiff's best reply; (ii) any arbitrary $e_P > 0$ is not Plaintiff's best reply. Similar steps give the result if we begin with supposing $e_P = 0$.

Step (i): For every $e_1 > 0$, Plaintiff emotional payoffs at $(e_P, e_D) = (e_1, 0)$ and at $(e_P, e_D) = (0, 0)$ are

$$\widehat{u}_P(e_1, 0) = \theta(e_1, 0)(1 - \xi)(1 + \upsilon + \widehat{\lambda}e_1) - e_1 + \xi \upsilon, \widehat{u}_P(0, 0) = \theta(0, 0)(1 - \xi)(1 + \upsilon) + \xi \upsilon.$$

By Assumption 2, $\theta(e_1, 0)$ does not depend on the exact value of $e_1 > 0$. Let $\tilde{\theta} = \theta(e_1, 0) \in \mathbb{R}$, for $e_1 > 0$. Fix any arbitrary (sufficiently small) number $e_1 > 0$ such that $e_1 < (1 - \xi)(1 + \upsilon)[\tilde{\theta} - \theta(0, 0)]$. The right-hand side is positive because $\xi < 1, \upsilon \ge 0$ and, by Assumption 8, $\tilde{\theta} > \theta(0, 0)$. As $\hat{\lambda}e_1\tilde{\theta} \ge 0$, then $e_1/(1 - \xi) < (1 + \upsilon)[\tilde{\theta} - \theta(0, 0)] \le (1 + \upsilon)[\tilde{\theta} - \theta(0, 0)] + \hat{\lambda}e_1\tilde{\theta}$. Some algebra then obtains $(1 - \xi)(1 + \upsilon + \hat{\lambda}e_1)\tilde{\theta} - e_1 > \theta(0, 0)(1 - \xi)(1 + \upsilon)$. Adding $\xi \upsilon$ on both sides then yields $\hat{u}_P(e_1, 0) > \hat{u}_P(0, 0)$. Hence $e_P = 0$ is not Plaintiff's best reply to Defendant playing $e_D = 0$.

Step (ii): Suppose, for a contradiction, there exists some $e_3 > 0$ such that choosing $e_P = e_3$ is a Plaintiff's best reply to Defendant choosing $e_D = 0$. Let e_4 be any arbitrary number satisfying $0 < e_4 < e_3$. Plaintiff's emotional payoffs at $(e_P, e_D) = (e_3, 0)$ and $(e_P, e_D) = (e_4, 0)$ are

$$\widehat{u}_P(e_3, 0) = \theta(e_3, 0)(1 - \xi)(1 + \upsilon + \widehat{\lambda}e_3) - e_3 + \xi \upsilon, \widehat{u}_P(e_4, 0)$$

= $\theta(e_4, 0)(1 - \xi)(1 + \upsilon + \widehat{\lambda}e_4) - e_4 + \xi \upsilon.$

Assumption 5 requires $(1-\xi)\widehat{\lambda} \leq 1$. Assumptions 2, 6, 8 imply $0 < \widehat{\theta} = \theta(e_3, 0) = \theta(e_4, 0) < 1$. Then $(1-\xi)\widehat{\lambda}\widehat{\theta} < 1$. Using this and $e_3 > e_4$, some algebra obtains $(1-\xi)\widehat{\lambda}e_4\widehat{\theta} - e_4 > (1-\xi)\widehat{\lambda}e_3\widehat{\theta} - e_3$. Adding $(1-\xi)(1+\upsilon)\widehat{\theta}$ on both sides yields $(1-\xi)(1+\upsilon+\widehat{\lambda}e_4)\widetilde{\theta} - e_4 > (1-\xi)(1+\upsilon+\widehat{\lambda}e_3)\widetilde{\theta} - e_3$. Adding $\xi \upsilon$ on both sides obtains $\widehat{u}_P(e_4, 0) > \widehat{u}_P(e_3, 0)$. Hence $e_P = e_3$ is not Plaintiff's best reply to Defendant playing $e_D = 0$, a contradiction.

Proof of Corollary 2 This proof establishes the result for $\mu > 0.5$. Similar steps establish the results for $\mu \le 0.5$.

Let $s = s^*$ and $\theta = \theta^*$. Lemma 1 and Proposition 1 prove that in the equilibrium of the Monetary Litigation Game, $s = (1 - \lambda\theta)/[1 - \lambda(1 - \theta)]$. Take the total derivative of both sides with respect to λ :

$$\frac{\partial s}{\partial \lambda} = \frac{\left(-\theta - \lambda \theta_s \frac{\partial s}{\partial \lambda}\right) [1 - \lambda(1 - \theta)] - \left(-(1 - \theta) + \lambda \theta_s \frac{\partial s}{\partial \lambda}\right) (1 - \lambda \theta)}{[1 - \lambda(1 - \theta)]^2}$$
$$[1 - \lambda(1 - \theta)]^2 \frac{\partial s}{\partial \lambda} = -\theta [1 - \lambda(1 - \theta)] + (1 - \theta)(1 - \lambda \theta) - \lambda [1 - \lambda(1 - \theta) + 1 - \lambda \theta] \theta_s \frac{\partial s}{\partial \lambda}$$
$$\Leftrightarrow \ s(1 - 2\theta) - \lambda(2 - \lambda) s \theta_s \frac{\partial s}{\partial \lambda} = s [1 - \lambda(1 - \theta)]^2 \frac{\partial s}{\partial \lambda} = (1 - \lambda \theta) [1 - \lambda(1 - \theta)] \frac{\partial s}{\partial \lambda}$$

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where the last equality uses Lemma 1. Then some algebra reveals

$$\frac{\partial s}{\partial \lambda} = \frac{s(1-2\theta)}{(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}$$
(21)

where part 9 of Lemma 2 proves $(1 - \lambda \theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s > 0$.

Now, obtain $\frac{d\theta}{d\lambda} = \theta_s \frac{ds}{d\lambda}$, where Lemma 2 proves $\theta_s < 0$. Let $\mu > 0.5$. Proposition 1 proves $s \le 1$. Part 1 of Lemma 2 and $\theta_s < 0$ together prove $\theta > 0.5$. From equation (21), $\theta > 0.5$ implies $\frac{\partial s}{\partial \lambda} < 0$. Then $\frac{d\theta}{d\lambda} > 0$.

Proof of Corollary 3 Let $s = s^*$ and $\theta = \theta^*$. Lemma 1 and Proposition 1 prove that in the equilibrium the Monetary Litigation Game, $s = (1 - \lambda\theta)/[1 - \lambda(1 - \theta)]$. Take the total derivative of both sides with respect to μ :

$$\frac{ds}{d\mu} = \frac{-\lambda \frac{d\theta}{d\mu} [1 - \lambda(1 - \theta)] - \lambda \frac{d\theta}{d\mu} (1 - \lambda\theta)}{[1 - \lambda(1 - \theta)]^2} = -\frac{\lambda(2 - \lambda)}{[1 - \lambda(1 - \theta)]^2} \frac{d\theta}{d\mu}$$

where taking the total derivative of θ with respect to μ reveals

$$\frac{d\theta}{d\mu} = \theta_s \frac{ds}{d\mu} + \frac{\partial\theta}{\partial\mu}.$$
(22)

Then a substitution exercise reveals

$$\frac{ds}{d\mu} \left[1 + \frac{\lambda(2-\lambda)\theta_s}{[1-\lambda(1-\theta)]^2} \right] = -\frac{\lambda(2-\lambda)}{[1-\lambda(1-\theta)]^2} \frac{\partial\theta}{\partial\mu}$$

$$\Leftrightarrow \frac{ds}{d\mu} \left[\frac{s[1-\lambda(1-\theta)]^2 + \lambda(2-\lambda)s\theta_s}{s[1-\lambda(1-\theta)]^2} \right] = -\frac{\lambda(2-\lambda)}{[1-\lambda(1-\theta)]^2} \frac{\partial\theta}{\partial\mu}$$

$$\Leftrightarrow \ \frac{ds}{d\mu}[(1-\lambda\theta)[1-\lambda(1-\theta)]+\lambda(2-\lambda)s\theta_s] = -\lambda(2-\lambda)s\frac{\partial\theta}{\partial\mu}$$

where the last step applies Lemma 1. Then some algebra reveals

$$\frac{ds}{d\mu} = \frac{-\lambda(2-\lambda)s\frac{\partial\theta}{\partial\mu}}{(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}$$
(23)

where part 9 of Lemma 2 proves $(1 - \lambda \theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s > 0$ and Assumption 3 holds $\frac{\partial \theta}{\partial \mu} > 0$. Hence $\frac{ds}{d\mu} \leq 0$, holding strictly if $\lambda > 0$. Then an application of the chain rule gives the results with respect to s^* .

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Now, using equations (22) and (23), some algebra reveals

$$\frac{d\theta}{d\mu} = \frac{\partial\theta}{\partial\mu} - \frac{\lambda(2-\lambda)s\theta_s \frac{\partial\theta}{\partial\mu}}{(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}$$
$$= \frac{\partial\theta}{\partial\mu} \left(1 - \frac{\lambda(2-\lambda)s\theta_s}{(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}\right)$$

which parts 6, 9 of Lemma 2 imply $\frac{d\theta}{d\mu} > 0$. An application of the chain rule gives the results with respect to θ^* .

Lemma 4 will be used to prove subsequent propositions and corollaries.

Lemma 4 Let $(e_P, e_D) = (e_P^*, e_D^*)$, the equilibrium characterized by Proposition 1. Let $s = s^*$ given by Lemma 1 and $\theta = \theta^*$ (Plaintiff's equilibrium probability of success). Denote $\omega = s/(1+s)^2$. The following holds:

1. $\lambda(2\theta - 1)/(2 - \lambda) = (1 - s)(1 + s), (1 - \lambda\theta)[1 - \lambda(1 - \theta)] = (2 - \lambda)^2 \omega.$ 2.

$$\mathbb{E}^* = \frac{-(2-\lambda)s\theta_s}{(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s} = -\left[\lambda + \frac{(2-\lambda)\omega}{s\theta_s}\right]^{-1}.$$

3.
$$\frac{d\omega}{d\lambda} = \frac{ds}{d\lambda}\omega(1-s)/[s(1+s)].$$

Proof of Lemma 4 Part 1: Using Lemma 1, some algebra will give these results.

Part 2: Apply Lemma 2 to system (20) to obtain $-[1 + \lambda \mathbb{E}^*]s\theta_s/e_P^* = 1 - \lambda\theta$, $-[1 + \lambda \mathbb{E}^*]s\theta_s/e_D^* = 1 - \lambda(1 - \theta)$. Using Lemma 1, some algebra will give the result. *Part* 3: Some algebra applying the chain rule gives the result.

Lemmas 5, 6 below provide technical results to facilitate the proof of Corollary 4.

Lemma 5 Function $\sigma(\cdot)$ defined by (11) exists and takes values in the interval (0, 0.5].

Proof of Lemma 5 This proof will (*i*) find the range of $\bar{\theta}(\cdot)$ given by (11); (*ii*) prove the existence and range of $\sigma(\cdot)$ given by (11).

Step (i): Some algebra using $0 \le \lambda \le 1$ reveals the values of function θ lay in [2/3, 0.75].

Step (ii): Suppose $\mu = 0.5$ and fix all other parameters. Proposition 1 establishes $\theta(s^*; 0.5) = \theta(1; 0.5) = 0.5$. Then $\frac{d}{d\mu}\theta(s^*; \mu) > 0$ from Corollary 3 and $0.5 < 2/3 \le \overline{\theta}(\lambda)$ from step (i) above together imply the existence of some $\mu' \in (0.5, 1]$ satisfying $0.5 < \theta(s^*; \mu') \le \overline{\theta}(\lambda)$. The value of σ is uniquely determined by the maximum of all such μ' . That $\mu \le 1$ further implies $\sigma \le 0.5$.

Lemma 6 below characterizes the necessary and sufficient condition for litigation expenditure to be increasing with the cost-shifting rule. Denote $\theta_{ss} = \frac{\partial^2 \theta}{\partial s^2}$ and define auxiliary functions α , $\beta : \mathbb{R}_{++} \to \mathbb{R}_{++}$ by

$$\alpha(s) = (1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s, \quad \beta(s) = \frac{(2 - \lambda)^2 s}{(1 + s)^2} + \lambda(2 - \lambda)s\theta_s$$

where $\theta \in \Theta(\{0\}, \{\lambda\})$, and μ, λ are exogenous parameters in functions α, β . Parts 6 and 9 of Lemma 2 imply $\alpha, \beta > 0$.

Lemma 6 Consider the equilibrium of the Monetary Litigation Game. Litigation expenditure \mathbb{E}^* is increasing with the cost-shifting rule λ if and only if

$$-\frac{(2\theta-1)s\theta_{ss}}{\theta_s} > (2\theta-1)\left[1-\frac{\lambda(2\theta-1)}{2-\lambda}\right] + \frac{\alpha(2-\lambda)s\theta_s}{(1-\lambda\theta)[1-\lambda(1-\theta)]} - \frac{\alpha}{2-\lambda}$$
(24)

where $s = s^*$ given by Lemma 1.

Proof of Lemma 6 Let $s = s^*$ given by Lemma 1. Part 2 of Lemma 4 reveals $\mathbb{E}^* = -[\lambda + (2 - \lambda)\omega/(s\theta_s)]^{-1}$. Differentiate both sides:

$$\frac{d\mathbb{E}^{*}}{d\lambda} = \left[1 + \frac{s\theta_{s}\left(-\omega + (2-\lambda)\frac{d\omega}{d\lambda}\right) - (2-\lambda)\omega(\theta_{s} + s\theta_{ss})\frac{ds}{d\lambda}}{s^{2}\theta_{s}^{2}}\right] \left(\lambda + \frac{(2-\lambda)\omega}{s\theta_{s}}\right)^{-2}$$
$$\frac{d\mathbb{E}^{*}}{d\lambda} = \left[s^{2}\theta_{s}^{2} + \left(-\omega + (2-\lambda)\frac{d\omega}{d\lambda}\right)s\theta_{s} - (\theta_{s} + s\theta_{ss})(2-\lambda)\omega\frac{ds}{d\lambda}\right]s^{-2}\theta_{s}^{-2}\mathbb{E}^{*2}$$
$$\frac{s^{2}\theta_{s}^{2}}{\mathbb{E}^{*2}}\frac{d\mathbb{E}^{*}}{d\lambda} = s^{2}\theta_{s}^{2} + \left(-\omega + (2-\lambda)\frac{d\omega}{d\lambda}\right)s\theta_{s} - (\theta_{s} + s\theta_{ss})(2-\lambda)\omega\frac{ds}{d\lambda}$$
$$\frac{s^{2}\theta_{s}^{2}}{\mathbb{E}^{*2}}\frac{d\mathbb{E}^{*}}{d\lambda} = s^{2}\theta_{s}^{2} - \omega s\theta_{s} + \frac{(2-\lambda)(1-s)\omega\theta_{s}}{(1+s)}\frac{ds}{d\lambda} - (\theta_{s} + s\theta_{ss})(2-\lambda)\omega\frac{ds}{d\lambda}$$
$$\frac{s^{2}\theta_{s}^{2}}{\mathbb{E}^{*2}}\frac{d\mathbb{E}^{*}}{d\lambda} = s^{2}\theta_{s}^{2} - \omega s\theta_{s} - \omega(2-\lambda)\frac{ds}{d\lambda}\left(\theta_{s} + s\theta_{ss} - \frac{(1-s)\theta_{s}}{1+s}\right)$$
(25)

where the second last equality uses Lemma 4, and equation (21) in the proof of Corollary 2. Then a substitution exercise using Lemma 4 and equation (25) gives the result.

Proof of Corollary 4 Let $s = s^*$ given by Lemma 1. Define a function $g(\mu, \lambda)$ by

$$g(\mu,\lambda) = s^2 \theta_s^2 - \omega s \theta_s - \omega (2-\lambda) \frac{ds}{d\lambda} \left(\theta_s + s \theta_{ss} - \frac{(1-s)\theta_s}{1+s} \right).$$
(26)

From equation (25), some algebra reveals that condition (24) is equivalent to $g(\mu, \lambda) > 0$. This proof will establish $g(\mu, \lambda) > 0$ in two cases: (*i*) $\mu = 0.5$; (*ii*) $0.5 < \mu \le 0.5 + \sigma(\lambda)$. Similar steps apply to $0.5 - \sigma(\lambda) \le \mu < 0.5$.

Case (*i*): Let $\mu = 0.5$. Corollary 2 proves $\frac{ds}{d\lambda} = 0$. Then $\theta_s < 0$ (from Lemma 2) implies $g(\mu, \lambda) > 0$.

Case (*ii*): Suppose $0.5 < \mu \le 0.5 + \sigma(\lambda)$. Use equation (21) and part 2 of Lemma 4 to obtain

$$-\omega(2-\lambda)\frac{\partial s}{\partial \lambda} = \frac{\omega(2-\lambda)s(2\theta-1)}{(1-\lambda\theta)(1-\lambda(1-\theta))+\lambda(2-\lambda)s\theta_s} = \frac{\omega(2-\lambda)s(2\theta-1)}{(2-\lambda)^2\omega+\lambda(2-\lambda)s\theta_s}$$
$$= \frac{s(2\theta-1)[(2-\lambda)\omega+\lambda s\theta_s-\lambda s\theta_s]}{(2-\lambda)[\omega(2-\lambda)+\lambda s\theta_s]} = \frac{s(2\theta-1)(1+\lambda\mathbb{E}^*)}{2-\lambda}.$$

Hence

$$g(\mu,\lambda) = s^2 \theta_s^2 - \omega s \theta_s + \frac{s(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} \left(s \theta_{ss} + \left(1 - \frac{1 - s}{1 + s}\right) \theta_s \right)$$
$$= s^2 \theta_s^2 - \omega s \theta_s + \frac{(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} \left(1 - \frac{1 - s}{1 + s}\right) s \theta_s + \frac{(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} s^2 \theta_{ss}$$

where $\theta > 0.5$ (from Proposition 1) and $s\theta_{ss} > \frac{2\lambda s\theta_s^2}{[1-\lambda(1-\theta)]}$ (from Lemma 2) imply

$$\begin{split} g(\mu,\lambda) &> s^2 \theta_s^2 - \omega s \theta_s + \frac{(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} \left(1 - \frac{1 - s}{1 + s}\right) s \theta_s \\ &+ \frac{(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} \left(\frac{2\lambda s^2 \theta_s^2}{[1 - \lambda(1 - \theta)]}\right) \\ &= s^2 \theta_s^2 - \omega s \theta_s + \frac{2s(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)(1 + s)} s \theta_s + \frac{(2\theta - 1)(1 + \lambda \mathbb{E}^*)}{(2 - \lambda)} \left(\frac{2\lambda s^2 \theta_s^2}{[1 - \lambda(1 - \theta)]}\right). \end{split}$$

Now, some algebra reveals

$$1 + \lambda \mathbb{E}^* = 1 - \frac{\lambda s \theta_s}{\omega (2 - \lambda) + \lambda s \theta_s} = \frac{\omega (2 - \lambda)}{\omega (2 - \lambda) + \lambda s \theta_s}$$
$$= \frac{-s \theta_s \omega (2 - \lambda)}{-s \theta_s [\omega (2 - \lambda) + \lambda s \theta_s]} = \frac{\omega (2 - \lambda) \mathbb{E}^*}{-s \theta_s}.$$

Hence

$$\begin{split} g(\mu,\lambda) > s^2 \theta_s^2 &- \omega s \theta_s + \frac{2s(2\theta-1)(1+\lambda \mathbb{E}^*)}{(2-\lambda)(1+s)} s \theta_s + \frac{(2\theta-1)\omega(2-\lambda)\mathbb{E}^*}{-s\theta_s(2-\lambda)} \left(\frac{2\lambda s^2 \theta_s^2}{[1-\lambda(1-\theta)]}\right) \\ &= s^2 \theta_s^2 - \omega s \theta_s + \frac{2s(2\theta-1)(1+\lambda \mathbb{E}^*)}{(2-\lambda)(1+s)} s \theta_s - \frac{2\omega(2\theta-1)\lambda \mathbb{E}^*}{1-\lambda(1-\theta)} s \theta_s \\ &= s^2 \theta_s^2 - \omega s \theta_s + \frac{2s(2\theta-1)}{(2-\lambda)(1+s)} s \theta_s \\ &+ 2s \theta_s \lambda \mathbb{E}^* (2\theta-1) \left[\frac{s}{(2-\lambda)(1+s)} - \frac{\omega}{1-\lambda(1-\theta)}\right] \end{split}$$

where some algebra using $s = (1 - \lambda \theta)/(1 - \lambda(1 - \theta))$ from Lemma 1 and the definition of $\omega = s/(1 + s)^2$ reveals

$$\frac{s}{(2-\lambda)(1+s)} = \frac{\omega(1+s)}{(2-\lambda)} = \frac{\omega(1-\lambda(1-\theta)+1-\lambda\theta)}{(2-\lambda)[1-\lambda(1-\theta)]} = \frac{\omega}{1-\lambda(1-\theta)}.$$

Hence, given $1/(1 + s) = (1 - \lambda(1 - \theta))/(2 - \lambda)$, some algebra obtains

$$g(\mu,\lambda) > s^2 \theta_s^2 - \omega s \theta_s + \frac{2s(2\theta-1)}{(2-\lambda)(1+s)} s \theta_s = s^2 \theta_s^2 - s \theta_s \left[\omega - \frac{2s(2\theta-1)}{(2-\lambda)(1+s)} \right]$$
$$= s^2 \theta_s^2 - s \theta_s \left[\frac{s}{(1+s)^2} - \frac{2s(2\theta-1)}{(2-\lambda)(1+s)} \right]$$

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$$=s^{2}\theta_{s}^{2} - \frac{s^{2}\theta_{s}}{(1+s)} \left[\frac{1}{1+s} - \frac{2(2\theta-1)}{2-\lambda} \right]$$
$$=s^{2}\theta_{s}^{2} - \frac{s^{2}\theta_{s}}{(1+s)} \left[\frac{1-\lambda(1-\theta)}{2-\lambda} - \frac{2(2\theta-1)}{2-\lambda} \right]$$
$$=s^{2}\theta_{s}^{2} - \frac{s^{2}\theta_{s}[3-\lambda-\theta(4-\lambda)]}{(1+s)(2-\lambda)}$$

where the definition of σ in (11) implies $3 - \lambda - \theta(4 - \lambda) \ge 0$. Then $\theta_s < 0$ (from Lemma 2) gives $g(\mu, \lambda) > 0$.

Proof of Proposition 4 Let $s = s^*$ given by Lemma 1. This proof will first prove the case of $\mu > 0.5$, and then the case of $\mu < 0.5$.

Case (*i*): Suppose $\mu > 0.5$. Then Proposition 1 proves that $s^* \leq 1$, holding strictly if $\lambda > 0$. The property $\theta_s < 0$ (from Lemma 2) implies $\theta(1; \mu) \leq \theta(s^*; \mu) = \theta^*$, where the weak inequality holds strictly if $\lambda > 0$. Then use Assumption 9 to obtain $\theta^* \geq \theta(1; \mu) = \mu$, where the weak inequality holds strictly if $\lambda > 0$.

Case (ii): Suppose $\mu < 0.5$. Use Assumptions 1, 9 to obtain:

$$1 - \mu = \theta(1; 1 - \mu) = 1 - \theta(1; \mu).$$
⁽²⁷⁾

Proposition 1 proves $s^* \ge 1$, holding strictly if $\lambda > 0$. The property $\theta_s < 0$ (from Lemma 2) implies $1 - \theta(1; \mu) \le 1 - \theta(s^*; \mu) = 1 - \theta^*$, where the weak inequality holds strictly if $\lambda > 0$. Then use (27) to obtain $1 - \theta^* \ge 1 - \theta(1; \mu) = 1 - \mu$, where the weak inequality holds strictly if $\lambda > 0$.

Proof of Corollary 5 An application of Proposition 4 and Corollary 2 gives the result.

Proof of Corollary 6 This proof will assume $\mu > 0.5$. The proof for the case of $\mu < 0.5$ follows from similar steps.

Let $\lambda = \widehat{\lambda}(1 - \xi)$. Use Proposition 2 and the chain rule to obtain

$$\frac{d}{d\xi}|\theta^*(\xi,\upsilon,\widehat{\lambda}) - \mu| = \frac{d}{d\xi}|\theta^*(0,0,\lambda) - \mu|$$
$$= \frac{d\lambda}{d\xi}\frac{d}{d\lambda}|\theta^*(0,0,\lambda) - \mu| = -\widehat{\lambda}\frac{d}{d\lambda}|\theta^*(0,0,\lambda) - \mu|.$$

An application of Corollary 5 gives the result.

Proof of Corollary 7 Define $\lambda = \hat{\lambda}(1 - \xi)$. An application of Proposition 2 and the chain rule obtains

$$\frac{d}{d\xi}s^*(\xi,\upsilon,\widehat{\lambda}) = \frac{d}{d\xi}s^*(0,0,\lambda) = \frac{d\lambda}{d\xi}\frac{d}{d\lambda}s^*(0,0,\lambda) = -\widehat{\lambda}\frac{d}{d\lambda}s^*(0,0,\lambda).$$

Part 1 follows from letting $\hat{\lambda} = 0$ or $\mu = 0.5$, and part 3 from Corollary 2.

Now, let $\mathbb{E}^*(\xi, \upsilon, \widehat{\lambda})$ denote the (equilibrium) litigation expenditure in the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \widehat{\lambda})$. From equation (9) and Proposition 2, obtain $\mathbb{E}^*(\xi, \upsilon, \widehat{\lambda}) = e_P^*(\xi, \upsilon, \widehat{\lambda}) + e_D^*(\xi, \upsilon, \widehat{\lambda})$ and then

$$\mathbb{E}^{*}(\xi, \upsilon, \widehat{\lambda}) = (1+\upsilon)(1-\xi)e_{P}^{*}(0, 0, \lambda) + (1+\upsilon)(1-\xi)e_{D}^{*}(0, 0, \lambda) = (1+\upsilon)(1-\xi)\mathbb{E}^{*}(0, 0, \lambda)$$
(28)

where $\mathbb{E}^*(0, 0, \lambda)$ is the (equilibrium) litigation expenditure in the Monetary Litigation Game $\mathbb{G}(0, 0, \lambda)$. Then apply the product rule and the chain rule to obtain:

$$\begin{aligned} \frac{d}{d\xi} \mathbb{E}^*(\xi, \upsilon, \widehat{\lambda}) &= \frac{d}{d\xi} \Big[(1+\upsilon)(1-\xi) \mathbb{E}^*(0, 0, \lambda) \Big] \\ &= (1+\upsilon) \Big[(1-\xi) \frac{d}{d\xi} \mathbb{E}^*(0, 0, \lambda) - \mathbb{E}^*(0, 0, \lambda) \Big] \\ &= (1+\upsilon) \Big[(1-\xi) \frac{d\lambda}{d\xi} \frac{d}{d\lambda} \mathbb{E}^*(0, 0, \lambda) - \mathbb{E}^*(0, 0, \lambda) \Big] \\ &= -(1+\upsilon) \Big[\mathbb{E}^*(0, 0, \lambda) + (1-\xi) \widehat{\lambda} \frac{d}{d\lambda} \mathbb{E}^*(0, 0, \lambda) \Big]. \end{aligned}$$

Then part 2 follows from letting $\widehat{\lambda} = 0$ and noting $\mathbb{E}^*(0, 0, \lambda) > 0$. Part 4 follows from Corollary 4.

Proof of Corollary 8 Part 1: Lemma 1 implies the equilibrium expenses ratio s^* used in Proposition 1 does not depend on v. The CSF θ also does not depend on v.

Part 2: Part 1 proves $\frac{d}{dv}\theta^*(\xi, v, \hat{\lambda}) = 0$. Then $\frac{d}{dv}|\theta^*(\xi, v, \hat{\lambda}) - 0.5| = \frac{d}{dv}|\theta^*(\xi, v, \hat{\lambda}) - \mu| = 0$.

Part 3: The proof for this part will establish the result for Plaintiff; similar steps give the result for Defendant.

Let $u_P^*(\xi, v, \hat{\lambda})$ denote Plaintiff's equilibrium monetary payoff when the emotional variables are ξ , v and the (unscaled) cost-shifting rule is $\hat{\lambda}$. Let $\theta^*(\xi, v, \hat{\lambda})$ denote her equilibrium probability of success given the triple $(\xi, v, \hat{\lambda})$. Using equations (1), (28) and Proposition 2, some algebra reveals

$$\begin{split} \frac{d}{d\upsilon} u_P^*(\xi,\upsilon,\widehat{\lambda}) &= \frac{d}{d\upsilon} \Big[\theta^*(\xi,\upsilon,\widehat{\lambda}) [1 + \widehat{\lambda} \mathbb{E}^*(\xi,\upsilon,\widehat{\lambda})] - e_P^*(\xi,\upsilon,\widehat{\lambda}) - \widehat{\lambda} e_D^*(\xi,\upsilon,\widehat{\lambda}) \Big] \\ &= \frac{d}{d\upsilon} \Big[\theta^*(\xi,\upsilon,\widehat{\lambda}) [1 + \widehat{\lambda} (1 + \upsilon) (1 - \xi) \mathbb{E}^* (0,0,\lambda)] \Big] \\ &- \frac{d}{d\upsilon} \Big[(1 + \upsilon) (1 - \xi) [e_P^*(0,0,\lambda) + \widehat{\lambda} e_D^*(0,0,\lambda)] \Big] \\ &= [1 + \widehat{\lambda} (1 + \upsilon) (1 - \xi) \mathbb{E}^* (0,0,\lambda)] \frac{d}{d\upsilon} \theta^*(\xi,\upsilon,\widehat{\lambda}) \\ &+ \theta^*(\xi,\upsilon,\widehat{\lambda}) \widehat{\lambda} (1 - \xi) \mathbb{E}^* (0,0,\lambda) \\ &- (1 - \xi) [e_P^*(0,0,\lambda) + \widehat{\lambda} e_D^*(0,0,\lambda)] \Big] \end{split}$$

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where part 1 reveals $\frac{d}{dv}\theta^*(\xi, \upsilon, \widehat{\lambda}) = 0$. Then

$$\begin{split} \frac{d}{d\upsilon} u_P^*(\xi,\upsilon,\widehat{\lambda}) = &\theta^*(\xi,\upsilon,\widehat{\lambda})\widehat{\lambda}(1-\xi)\mathbb{E}^*(0,0,\lambda) - (1-\xi)[e_P^*(0,0,\lambda) + \widehat{\lambda}e_D^*(0,0,\lambda)] \\ = &-(1-\xi)(1-\theta^*(\xi,\upsilon,\widehat{\lambda})\widehat{\lambda})e_P^*(0,0,\lambda) - \widehat{\lambda}(1-\xi)(1-\theta^*(\xi,\upsilon,\widehat{\lambda}))e_D^*(0,0,\lambda). \end{split}$$

Then $\xi < 1, 1-\theta^*(\xi, \upsilon, \widehat{\lambda})\widehat{\lambda} > 0$ and $1-\theta^*(\xi, \upsilon, \widehat{\lambda}) > 0$ imply $\frac{d}{d\upsilon}u_P^*(\xi, \upsilon, \widehat{\lambda}) < 0$. *Part* 4: The result follows from differentiating both sides of equation (28) with respect to υ .

B Appendix: Calculations for illustrative CSFs

This Appendix offers calculations for the illustrative CSFs, θ_T and θ_W . Denote $s = e_D/e_P$ if $e_P, e_D > 0$.

The Tullock CSF θ_T given by (8) satisfies the following properties:

$$\begin{split} &\frac{\partial \theta_T}{\partial e_P} = \frac{\gamma \mu (1-\mu) e_P^{\gamma-1} e_D^{\gamma}}{[\mu e_P^{\gamma} + (1-\mu) e_D^{\gamma}]^2} \quad \frac{\partial^2 \theta_T}{\partial e_P^2} \\ &= \frac{\gamma \mu (1-\mu) e_P^{\gamma-2} e_D^{\gamma} [(\gamma-1)(1-\mu) e_D^{\gamma} - (\gamma+1) \mu e_P^{\gamma}]}{[\mu e_P^{\gamma} + (1-\mu) e_D^{\gamma}]^3} \quad \frac{\partial \theta_T}{\partial s} = \frac{-\mu (1-\mu) \gamma s^{\gamma-1}}{[\mu + (1-\mu) s^{\gamma}]^2} \\ &\frac{\partial^2 \theta_T}{\partial s^2} = \frac{\mu (1-\mu) \gamma s^{\gamma-2} [(1-\mu) (\gamma+1) s^{\gamma} + (1-\gamma) \mu]}{[\mu + (1-\mu) s^{\gamma}]^3} \\ &\frac{\partial \theta_T}{\partial s} + s \frac{\partial^2 \theta_T}{\partial s^2} = \frac{\mu (1-\mu) \gamma^2 s^{\gamma-1} [(1-\mu) s^{\gamma} - \mu]}{[\mu + (1-\mu) s^{\gamma}]^3}. \end{split}$$

Given θ_T with $\gamma = 1$ and $\xi = \upsilon = 0$, the equilibrium expenses ratio is

$$s^* = \frac{1 - 2\mu + \sqrt{(1 - 2\mu)^2 + 4\mu(1 - \mu)(1 - \lambda)^2}}{2(1 - \mu)(1 - \lambda)}$$

The Plott CSF θ_W given by (10) satisfies the following properties:

$$\frac{\partial \theta_W}{\partial e_P} = \frac{(1-\eta)\gamma e_P^{\gamma-1} e_D^{\gamma}}{[e_P^{\gamma} + e_D^{\gamma}]^2}$$
$$\frac{\partial^2 \theta_W}{\partial e_P^2} = \frac{(1-\eta)\gamma e_P^{\gamma-2} e_D^{\gamma} [(\gamma-1)e_D^{\gamma} - (\gamma+1)e_P^{\gamma}]}{[e_P^{\gamma} + e_D^{\gamma}]^3}$$
$$\frac{\partial \theta_W}{\partial s} = \frac{-(1-\eta)\gamma s^{\gamma-1}}{(1+s^{\gamma})^2}$$

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$$\frac{\partial^2 \theta_W}{\partial s^2} = \frac{(1-\eta)\gamma s^{\gamma-2}[(1+\gamma)s^{\gamma}+1-\gamma]}{(1+s^{\gamma})^3}$$
$$\frac{\partial \theta_W}{\partial s} + s \frac{\partial^2 \theta_W}{\partial s^2} = \frac{(1-\eta)\gamma^2 s^{\gamma-1}[s^{\gamma}-1]}{(1+s)^3}.$$

Given θ_W with $\gamma = 1$ and $\xi = \upsilon = 0$, the equilibrium expenses ratio is $s^* = [1 - \lambda + \lambda \eta (1 - \mu)]/[1 - \lambda + \lambda \eta \mu]$.

Acknowledgements We acknowledge the comments given by Fabio Araujo, Ted Bergstrom, Subhasish Chowdhury, Richard Cornes, Simona Fabrizi, Luciana Fiorini, Tim Friehe, Simon Grant, Richard Holden, Tim Kam, Shay Lavie, Steffen Lippert, Leandro Magnusson, George Mailath, Rogerio Mazali, Jonathan Nash, Francesco Parisi, Paul Pecorino, Omer Pelled, John Quiggin, Maria Racionero, Martin Richardson, Larry Samuelson, Kathryn Spier, James Taylor, Joshua Teitelbaum, Abraham Wickelgren, Kathy Zeiler and seminar participants at the 2018 Annual Meeting of the American Law and Economics Association, the 2017 Annual Meeting of the Canadian Law and Economic Association, the 2016 Australian Law and Economic Conference, the 2014 Australasian Economic Theory Workshop, the 2014 Econometric Society Australasian Meeting, and workshops at the Australian National University and the Universities of Auckland and Western Australia. We benefited greatly from the insightful comments of an anonymous co-editor and two anonymous reviewers. Chen acknowledges the support of an Australian Government Research Training Program Scholarship. All mistakes are our own responsibility.

Funding Open Access funding enabled and organized by CAUL and its Member Institutions.

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