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Path dependence in an evolving system: a modeling perspective

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Abstract

In recent years, path dependence has gained increasing scientific attention in many disciplines, leading to various new concepts and notations, such as path creation or path plasticity. However, if mathematical arguments are used, they are based on the early works by Brian W. Arthur and Paul A. David, usually referring to the mathematical concept of ergodicity. We extend their mathematical framework and develop a graphical representation of systems that allows for a metaphorical discussion of system behaviors beyond the original cases, especially in evolving systems, and the inclusion of the recently developed concepts within path dependence. Visualizations are used to explain the definition and characteristics of seven types of path dependence: lock-in, path-breaking, path-furrowing, path plasticity, path formation, path creation, and path selection. Although these visualizations are explained verbally and can be understood without a mathematical expertise, a mathematical model is used to generate them. The deduction of the metaphorical concept from a mathematical model guarantees the completeness of the identified processes and the rigor in their categorization as well as the identification of respective characteristics for their distinction. However, the aim of the paper is to provide an illustrative concept that allows researchers to classify and structure the various path-dependent processes they observe in their application. While five of the identified processes are in line with concepts from the literature and are defined accordingly, we also detect a sixth process that is new to the literature so far: path-furrowing. Moreover, slightly deviating from the literature, we define path selection as the possibility to choose a path intentionally, thereby focusing on the mindful choice of options.

Keywords Path dependence · Paths · Lock-in · Path-breaking · Path-furrowing · Path plasticity · Path formation · Path creation · Path selection · Evolving systems

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1 Introduction

In recent years, path dependence has received increasing scientific attention, especially in the geographic and economic literature (e.g., Boschma and Frenken 2006; Garud et al. 2010; Martin and Sunley 2015). Paul A. David (1985) originally and widely introduced the concept of path dependence into the economic literature. In the following years, several researchers applied the concept mainly on topics related to diminishing and increasing returns, network externalities, and explanations of lock-in states (e.g., Arthur et al. 1986, 1989). However, the range of applications increased tremendously when researchers began to use the concept to explain regional, technological, environmental, political, and organizational mechanisms and developments. Furthermore, the introduction of related concepts such as path creation, path plasticity, or exiting a path reflects the progressively diverse and detailed application of the concept within various scientific fields.

Paul A. David (2007) developed the concept of path dependence based on ergodicity, a mathematical characteristic of stochastic processes defined on a given state space of a system. While ergodicity and the original mathematical formulations are adequate in the context of network externalities and the explanation of increasing returns, they do not fit all the dynamics in more complex and evolving systems, such as regional economic systems or technological systems. Although the earlier papers by Paul David and Brian Arthur (David 1985, 1997; Arthur et al. 1983) provided a more detailed understanding of processes, their illustrations had been, in principle, limited to cases with one state-defining dimension, leading to the dissemination of one clearly defined concept of path dependence. The following applications of path dependence to a huge variety of situations and processes led to deviations from the original concept by using verbal definitions of path dependence and developing various related notions (e.g., North 1990; Pierson 2000; Vergne and Durand 2010; Djelic and Quack 2007). This is why the paper at hand follows the intellectual tradition of Paul A. David and develops a more general mathematical model representing especially evolving systems and involving also the various recently developed concepts. Even though mathematical approaches have their shortcomings, they are still valuable in supporting the understanding of the underlying mechanisms and bringing the circulating definitions and conceptions of path dependence down to a common denominator (Bassanini and Dosi 2001). Nevertheless, the mathematical model is only a mean to reach the purpose of this paper and is therefore presented in the Appendix. Our aim is to provide a metaphorical concept in the form of illustrations that can be used by researchers to classify and structure the path-dependent processes they observe.

Paul David and especially Brian Arthur (Arthur et al. 1983, 1987; Arthur 1988) have already used illustrations of their concepts. However, these illustrations have been two-dimensional, limiting the state space to one dimension. The important step to include all possible path-dependent processes that are discussed in the literature, is the use of three-dimensional illustrations and a distinction between two state variables with different characteristics, namely uni-directional variables and multi-directional variables. The interplay of these two kinds of variables is essential for some path-dependent processes and their distinction. By clearly showing their importance and the implications, we contribute to an extended understanding for the processuality of path dependence in evolving state spaces. This is done in the paper at hand and allows to include all commonly known processes that have been detected in the path dependence literature in the last 20 years, such as lock-in, path-breaking, path plasticity, path formation, and path creation. Based on a mathematical model we develop a graphical representation of systems which includes all those processes mentioned before. It allows for clearer definitions of these processes and, especially, shows their differences. Moreover, it allows to identify possible proceedings, which leads to the detection of a so far undefined process within path dependence: path-furrowing. Each of the processes is discussed separately, and hints on their detection and application are given.

We also comprehensively discuss the aspects of randomness and endogeneity in path-dependent processes. While the paper does not provide a complete exemplification of the model, it still gives insights about how the corresponding existing processes can look like. This paper intends to provide a more systematic and mathematical-based perspective on the different types of path-dependent processes and thereby set the ground for further definitions and applications.

The remainder of the paper proceeds as follows: The next section contains a presentation of the current literature on path dependence. In this chapter we also highlight the necessity of a new understanding of the concept and raise some fundamental issues. In section three, we develop and explain our basic concept to depict path dependence accordingly. The theoretical framework is described here, and basic considerations are mentioned. For those readers who are particularly interested in our model's mathematical background, the Appendix is of specific interest. Section four contains the deduction of the various kinds of path-dependent processes, their definitions, the discussion of the possible developments and their relations to the state of the art. Finally, the paper concludes in section five.

2 Theoretical background

2.1 Path dependence

2.1.1 The beginnings

Brian Arthur and Paul David laid the foundation for all path dependence related research. In 1983, Brian Arthur developed a general urn scheme of technology choice based on people's preference to choose the same technology others have chosen before (Arthur et al. 1983). In his working paper on competing technologies (Arthur 1983), which was basically an early version of his work from 1989 (Arthur 1989), Arthur describes the importance of small, sometimes unpredictable events for the eventual outcome. Two years later, in 1985, David draws the development of the well-known "QWERTY"-case, which is frequently used in path dependence related papers as an example, and thereby widely introduced the concept of path dependence into the economic literature. Paul A. David (1985) applied the concept of path dependence to explain why the actual keyboard structure became "'locked in' as the dominant keyboard arrangement" (David 1985, p. 334) and has not changed, although it is well known that a different sorting of letters would make typing faster. In his later clarifications, David explains that path dependence is more than technological alterations: it also influences the development of other (also social) systems (David 2001). Brian Arthur and his co-authors Ermoliev and Kaniovski used mathematical models to describe the adoption of technologies in the market and the role of path dependence (Arthur et al. 1983, 1986, 1987; Arthur 1989). In all these works, path dependence is strongly related to the concept of lock-in, meaning that a system reaches one possible stable state, while other, perhaps "better" alternatives exist. Following Arthur, it is challenging to identify the (small) events in history that cause the decision to follow one or another path at the fork of the road. On the one side, it is hard to, ex-ante, determine the events which resulted in change. On the other side, it is difficult to track the exact causality of the events involved ex-post (Arthur 1983). Arthur, Ermoliev, and Kaniovski sum up three principles of path dependence: It is nearly impossible to accurately predict the dominant technology ex-ante. This dominant technology does not necessarily have to be the most efficient one, and increasing self-reinforcing structures decrease the chances of changes (Arthur et al. 1987). Independent of whether path dependence leads to an inferior state, the definition of path dependence highlights that the considered dynamics have several potential outcomes (or outcome distributions) and that one of them is reached due to some initial developments. In the words of Paul A. David (2007, p. 98, see also 2001, 1997): "a path dependent stochastic system is one possessing an asymptotic distribution that evolves as a consequence (function) of the process's own history".

2.1.2 Ergodicity and other characteristics of path dependence

In line with this, the original concepts of path dependence tend to refer to ergodicity, more precisely to non-ergodicity, a mathematical characteristic of stochastic processes: "Stochastic processes like that do not converge automatically to a fixed-point distribution of outcomes, and are called *non-ergodic*" (David 1985, p. 332 [italics original]). Arthur (1989, p. 118) also equates "ergodic" with "not path-dependent" implying the same definition. To make the meaning of ergodicity clearer, Arthur et al. (1987) differentiate between different processes:

> "We could usefully distinguish between processes that have a single stable fixed point, so that a unique structure or limit must emerge, and processes that have more than one reachable stable fixed point, where structure is 'selected' partly randomly. The former we can call *ergodic*—there is one possible outcome, and perturbations 'wash away'. The latter *are non-ergodic*—there are multiple outcomes and early perturbations become all-important in the 'selection' of structure" (Arthur et al. 1987, p. 301 [italics original]).

Many other early studies in economics and geography have also followed David's and Arthur's definitions of path dependence. Most of them used stochastic models.

Considering these studies and the aforementioned concept of path dependence, we find the following distinct defining parts of path dependence:

1. There is a considered system with a defined set of possible states and a stochastic process determining its dynamics.
2. There is more than one state (or distribution of states) that the system might converge to and it is not predetermined which of these states is reached (non-ergodicity).

Another famous example for a path-dependent stochastic model is the mathematical modeling of decision-making rules based on an urn scheme with balls of different colors (Arthur et al. 1983). Arthur used this concept by interpreting balls as companies and their colors as regions where the company might want to establish (Arthur 1990). This approach paved the way for the concept's application in economic geography. By using several examples like propulsion technology, programming languages, or television system, Arthur, Ermoliev and Kaniovski show that former choices influence the following choices, for regions as for technologies: "if one region by chance gets off to a good start, its attractiveness and the probability that it will be chosen becomes enhanced. Further firms may then choose this region; it becomes yet more attractive" (Arthur et al. 1987, p. 295–296).

2.1.3 Recently developed concepts: an overview

As already mentioned, several concepts evolved from the original definitions of path dependence and encroached upon plenty kinds of research fields. Table 1 gives an overview of the various path dependence concepts and their according understandings. The connection of these concepts to our approach is elaborated in Sect. 4, where we analyze the different path-dependent processes.

2.2 Necessity of a new understanding

While the original understanding and definition of path dependence by Paul A. David (1985, 2007) and Brian Arthur (1989) is given, the recent literature shows that many further definitions have been derived that deviate from the original understanding. As various as the different definitions of path dependence are (see Sect. 2.1), so are the manifold usages of the related concepts.

The main issue of the original definition is the connection to the non-ergodicity of the system. As mentioned above, ergodicity is mathematically defined on a given set of states of a system and requires that any state is reachable independent of what happened in the past. Subsequently, non-ergodicity means that transitions to certain (past) states are not possible. Let us consider the original topic addressed by Paul A. David: the keyboard. If we assume that there are two possible keyboard designs (CLIO and QWERTY), then the system's state is given by the share of the population that uses the QWERTY keyboard. Hence, the set of possible states is given by all values from 0 to 100%. Independent of whatever stochastic process is defined,

Table 1 Overview of different path-dependent processes

Concept	Author	Understanding
Path dependence	David (1985)	Actions in the past influence events in the presence. A path-dependent system is strongly connected to its own history
	Arthur (1989)	Attractiveness of a technology increases the more people use it—former choices of others influence the present choices
	Sydow et al. (2009), Vergne and Durand (2010), Strambach and Halkier (2013), Heimeriks and Boschma (2014)	Expansion of the concept to organizational and institutional structures (and their respective lock-in), and on knowledge production (place dependence of path dependence within knowledge dynamics)
	Bassanini and Dosi (2001)	Differentiation between small and big events influencing the path dependence as well as differentiation between weak and strong forms of path dependence
	Pierson (2000), Martin and Sunley (2006)	Expansion of the concept to political and social sciences
(Non-)Ergodicity	David (1985)	Non-ergodic = no automatically convergence to a fixed-point distribution Ergodic = not path-dependent
	Arthur et al. (1987)	Ergodic = a unique structure or limit must be reached Non-ergodic = several possible outcomes
Lock-in	Vergne and Durand (2010)	"Hard-to-escape situation" (p. 743)
	Grabher (1993), David (2001), Garud et al. (2010), Martin (2010), Martin and Sunley (2010), Puffert (2019)	Lock-in as a central element to path dependence that can cause temporal stability and/or stagnation and sometimes needs an exogenous shock to be dissolved
Path-breaking	Sydow et al. (2009), Sydow et al. (2012)	Restoration of a state of choice with at least one real alternative
	Meyer and Schubert (2007)	Path-breaking: intentional termination of the path
Path dissolution		Path dissolution: ending without decisive action
Lock-out	Arthur (1988)	Lock-out happens simultaneously to locking-in into another path
De-locking	Bassanini and Dosi (2001)	Leaving a lock-in either through change of existing knowledge or due to an exogenous shock
	Strambach and Halkier (2013)	"De-locking the path by breaking it and/or creating a new one" (p. 1)

Table 1 (continued)

Concept	Author	Understanding
Path creation	Garud and Karnøe (2001)	Actors are aware of their possibilities and their mindful actions shape the future paths
	Martin and Sunley (2010), Meyer and Schubert (2007), Hirsch and Gillespie (2001)	Processuality of ongoing interaction, multi-layered, deliberate development
	Stack and Gartland (2003)	Focus on firms: Firms actively design their environments and shape their futures
Path constitution	Meyer and Schubert (2007)	Path constitution = path creation + path dependence, between completely unplanned and completely controlled processes
	Sydow et al. (2012)	Framework to analyze path dependence and path creation
Path generation	Djelic and Quack (2007)	Path generation is a combination of path creation + path plasticity
Path plasticity	Strambach and Storz (2008), Strambach (2010), Strambach and Halkier (2013), Strambach and Klement (2013), Notteboom et al. (2013), Martin and Sunley (2015)	Change within a path due to variability, alteration, and continuous change (including incremental institutional innovations), no exogenous shock is needed for slight changes

one will end up with one of the following two situations: In the long run, the stochastic process will converge to a stable distribution of state probabilities (ergodic) independent of the starting point and the random events that occurred. Or there is more than one stable state (or distribution) that the process can converge to, and the starting point and the path become decisive. Such a situation allows for a simple negative definition of path dependence as non-ergodicity of the system as done by Paul A. David (2007, p. 97):

> “One route to a precise definition of the term starts by distinguishing between path dependent dynamics, and all the rest. The latter appropriately are labeled ‘‘path independent’’, because their dynamics guarantees that they converge to a unique, globally stable equilibrium configuration regardless of where they started, or how they approached that eventual outcome”.

As a show case, let us now consider not the keyboard but the word processor that we use and ask again the simple question of what share of the population uses which word processor. The situation is different since nobody will move back to Word 2.0. Consequently, new versions of word processors will continue to appear. This causes two problems considering the original path dependence concept. First, it is difficult or impossible to define a set of states for the future. New options can arise that are often unforeseeable. Nevertheless, this does not apply to all systems. For example, the ways we can order the keys on a keyboard are overall limited and fixed, the same holds for the possible outcomes of coin-tossing (Arthur et al. 1987). Still, this holds for many issues that are implemented in fields such as economics and geography. However, it might be possible to abstractly define adequate open state spaces in many cases (e.g., with the help of describing variables that are not limited in their values). Second, due to technological development and social and cultural evolution, most processes are not reversible in real life. Hence, events are permanently occurring that eliminate states from the set of still possible states (such as a new word processor will eliminate the state of most of the people using the old one). If the set of possible states is enlarged by new appearing options and reduced by options, there cannot be a stable state or distribution. Hence, applying the above-quoted negative definition by Paul A. David (2007) would imply that all processes with such changing sets of possible states are path-dependent. And although David (1985) already mentioned evolving state spaces in his first paper on path dependence, the concept itself was more in the focus than the decidedly examination of the occurring state spaces. That’s why further elaboration is needed.

Basically, this implies that we should distinguish between two kinds of systems: (1) systems with a stable state space (such as the keyboard example) where path dependence can be well defined by non-ergodicity and (2) systems that evolve due to permanent changes of the state space by possible states being removed and added. The latter type of system is automatically non-ergodic and therefore path-dependent, considering the original definition. However, these systems show different kinds of dynamics connected to path dependence. Hence, it is worth taking a more detailed look and defining various subcategories of path dependence for such systems. The literature has already developed in this direction with many concepts being proposed by various authors (see Sect. 2.1).

2.3 Fundamental issues

Before entering the endeavor to model path-dependent processes, two issues must be clarified because they resulted in both discussions and confusion in the literature on path dependence: *randomness* and *endogeneity*.

2.3.1 Randomness

In the literature, the relevance of randomness in path dependence has been discussed widely. While the early literature is based on small random events that trigger the development into a certain direction—very well visible in the urn scheme models by Arthur (Arthur et al. 1983; Arthur 1990)—the more recent literature emphasizes the intentionality of action responsible for the further development, especially in the field of path creation (Garud and Karnøe 2001). Bassanini and Dosi (2001) proposed distinguishing between small frequent and large rare events, implying that processes with different degrees of randomness are involved.

All authors in the field of path dependence agree that events or decisions can be pivotal for the further development of systems. Therefore, the question of how these events and decisions form is crucial. Randomness in the context of human behavior is a question of perspective: One could take the perspective of an outside observer or modeler and argue that without further information about a person's history and way of thinking we are often not able to predict behavior. In this context, Garud et al. (2010) highlight the relevance of events a priori unknown to the observer. We could also take the perspective that all, or at least most, action is intentional and knowing all motives, characteristics, and experiences of a person would allow us to predict this person's action. This analysis is often pursued in retrospect to understand the development.

Instead of taking one of those perspectives, we follow Bassanini and Dosi (2001) and distinguish between actions of different degrees of randomness. This is necessary to understand the different kinds of path dependence examined in Sect. 4. We assume that a system that involves human action is analyzed and that the range of the system is clearly defined (see discussion of endogeneity below). Consequently, we distinguish three kinds of randomness in actions, which all occur simultaneously in larger systems:

1. *Random actions*: Some actions are taken by mistake, without consideration, or as a choice with indifference between options. Sometimes humans simply do not care or cannot predict the outcome and choose an option by chance. This can be rationalized ex-post but we define these decisions as random actions and assume that to a certain extent, they are happening all the time in all systems. For example, they occur frequently in technological developments where researchers state that there have been different options, and nobody was able to predict which option would eventually work. They simply started to test one by one. Another

example are business ideas. Skills and experiences make it more likely that certain people develop certain ideas but random events impact the generation process of ideas.

2. *Intentionally deviating actions*: Some actions are surprising from an outside perspective because they are not in line with the expected or most likely action but they are intentionally taken. For example, in the VHS and Betamax competition, which were two competing video recording systems in the 1970s and 80 s, the firms that provided the VHS system took right decisions and made some crucial alliances that influenced the further development (Liebowitz and Margolis 1995; Puffert 2019). From a modeling perspective, such actions cannot be predicted ex-ante, but it can be understood ex-post.
3. *Random change of external circumstances*: A third source of randomness from the perspective of the system's development are changes of external circumstances, which may alter the processes, actions, and dynamics of the system. In principle, we do not have to discuss the randomness of such external changes because we defined our system and declared them as external (see the following discussion on endogeneity). These random changes include, for example, natural disasters as well as predictable changes on a higher system level such as trends in population attitudes.

It is true that other classifications could be taken and that the types above are described more distinctively than they sometimes are. Nevertheless, we believe that this distinction is helpful to understand different types of path dependence.

2.3.2 Endogeneity

For modeling the dynamics of a system, it is important to define which processes are explicitly modeled, hence endogenous, and which processes are seen as exogenous and therefore not calculated. This means that the boundaries of the modeled system must be declared. In the context of path dependence, Sydow et al. (2012, p. 159) talk about the "focal level of analysis". They argue that this level of analysis is interrelated "to surrounding levels of analyses that are more micro and macro" (Sydow et al. 2012, p. 159). Clearly, explicitly defining the levels of analysis within the model is relevant because the boundaries of the system are set by the modeler, and the focus on a certain level changes the perspective. There is no given or optimal definition of the system. The modeler decides, which processes are modeled. This has implications on the definition of the path-dependent processes within the model. We will explain this later when we define the different kinds of path dependencies. For the moment, we only want to stress the fact that the modeler decides which processes are endogenous (included in the model), and which are exogenous.

An additional aspect that should be mentioned is related to what Sydow et al. (2012, p. 159) call the "more micro and macro" levels within their idea of level interrelatedness. This means that on the one hand there are levels above the system that are not included in the model but influence its dynamics (macro). This can be, e.g., global trends or policy decisions from the government above the system level. Usually, as an analytical tool, we do not consider the whole world and therefore restrict the analysis to a limited system. We then consider dynamics above the system as exogenous influences. However, in principle, it is possible to enlarge the system and include and explain the global dynamics above.

On the other hand, some levels are found on the micro spectrum. As previously mentioned, we consider systems that are based on human actions. This implies that the system analysis contains many individuals or actors and their associated interactions or interrelatednesses, which in the end cause path dependence. Hence, the micro level can be considered as the level of individual behavior. Again, we could argue that this micro-level could be, at least partly, included in the model. However, this would mean that we have to model and, thus, predict the involved people's action. Behavioral science has shown that we can understand—at least in most cases—actions ex-post but that we are not able to predict actions exactly. The use of probabilities for the various actions is what can be done and thus is usually done in mathematical models.

In the context of path dependence this is an important issue. Developments in the form of a path are usually assumed to be expectable, natural, or even unavoidable. If a path is abandoned due to the action(s) of one or a few people, the question arises whether these actions could have been expected or not. If they could have been expected, it would be a continuation of the path. But if we argue that one can only predict the probabilities of actions (and not the actions themselves), it appears that all actions are possible. We will discuss this further below.

3 Theoretical framework

3.1 Basic considerations

The theoretical framework we offer is completely based on abstract graphical representations of systems, so that it can be understood and applied without any mathematical background. We provide verbal descriptions of the processes and aspects included in the model and explain the thereby arising theoretical framework in the following.

Our model is based on describing the state of the considered system by a set of variables. The state of the system is assumed to change permanently due to two different kinds of processes: (1) There is a causal relationship between the variables' dynamics, leading to inherent dynamics of the system, which are built into the model. An example for those dynamics is that the use of the QWERTY layout by most of the customers causes further customers to also use this layout. (2) There are random changes in the system that do not follow the above causal logic, e.g., customers choosing a different layout, although the QWERTY layout dominates the market, due to personal reasons, insufficient information, or by mistake.

In addition to those two processes, there are (unforeseeable) changes of the state of the system due to external events or shocks, e.g., if the government decides in favor of one keyboard layout and prohibits all other layouts. In contrast to the two processes above, such changes are not included in the model as endogenous processes. Hence, all following graphs represent the endogenous dynamics of the system, excluding external events and shocks. Of course, what is seen as external and as shock depends on the definition of the analyzed system as discussed above (Sect. 2.3).

An important feature of our model is the differentiation of uni-directional and multi-directional variables (see Sects. 3.2 and 3.3). Uni-directional variables characterize aspects of the system that are changed by the causal endogenous dynamics always in one direction independent of the actual state. An example is the use of different versions of a product, like the Word processor: The use of the newest, and probably best version will always increase while the use of older versions decreases. Multi-directional variables are those that represent characteristics of the system that are changed in different directions by the causal endogenous dynamics depending on different actual states. E.g., the use of the QWERTY layout increases the more people use it and decreases the more people use another layout. This part of our approach is in line with the original understandings of path dependence.

Considering different kinds of variables is the main novelty in our approach. While there are many papers in which the underlying processes are mentioned, discussed, or used, a general discussion based on the differentiation of these two kinds of variables (or processes) is missing. We argue that such a differentiation is crucial to understand path-dependent processes.

To sum up, our approach builds on two central aspects: (1) The distinction between uni- and multi-directional variables and (2) the inclusion of causal dynamics as well as randomness.

In the Appendix, a general mathematical model that includes these aspects is developed, based on a Fokker–Planck equation. Although there might be applications that are difficult or impossible to be represented exactly by this model, the model is sufficiently flexible to examine the fundamental features of path-dependent systems. Especially, it allows to depict the various path-dependent processes within three-dimensional graphs. This mathematical model is used for two reasons: First, the model provides us with a tool to rigorously check which characteristics, conditions, and relationships lead to the corresponding processes and outcomes. This is important to draw profound implications. Second, the literature on the Fokker–Planck equation applies a so-called potential function (see Sect. 3.4), which can be easily depicted and understood, and additionally is in line with some original metaphors in the path dependence literature. We will use these potential functions in the further course to show and discuss the various path-dependent processes.

3.2 Uni-directional variables

As already mentioned above, uni-directional variables characterize aspects of the system that are changed by the causal endogenous dynamics always in one direction independent of the actual state. The first association might be that these variables are irreversible, but randomness applies also to these variables. This means that developments in the opposite direction are possible. However, the average development always leads in one direction. Hence, these variables are not perfectly irreversible but reaching former states becomes more and more unlikely with time passing because the system distances itself more and more from them. We might call these variables weakly reversible because the latest changes might be reversed due to randomness but in principle the variables will not turn back to states from further in the past. This also implies that applying the original mathematical tools, such as studying multiple equilibrium states, is not helpful in this respect. That being the case, the system will never reach an equilibrium state due to the permanent development in the uni-directional variable.

Often uni-directional variables are related to technological or cultural developments. In applications they can be identified by the agreement of most of the involved people on a certain advancement or standard. They address aspects where no one has an incentive to move "back" (Word processor example). This means that the causal processes have a clear direction, although they might be obstructed by situations, such as lock-ins, or might be reversed slightly by random events, such as mistakes.

3.3 Multi-directional variables

Multi-directional variables are those that represent characteristics of the system that can change in different directions induced by the causal endogenous dynamics. These variables are reversible, and it can be assumed that their state space is fixed. Thus, stationary distributions can, in principle, be calculated.

In applications, these variables are often shares of populations (people, firms, etc.) that show a certain aligned behavior, e.g., buy a certain product, use a certain technology, or are located in a certain region. If there are only two possible options, we can abstractly represent the state of the system by one variable ranging from 0 to 1 and denoting the share of the population that uses option 1. If there are more than two options, more than one dimension is required to describe the state of the system. However, these variables can also represent other kinds of system characteristics, such as characteristics of institutions, organizations (e.g., structures within firms), or (some) political decisions. In principle, all characteristics of the system that can be changed back and forth, although this might be unlikely (for example due to lock-ins), are multi-directional variables.

For convenience the uni-directional variable can be interpreted as time. In many applications this does not influence the interpretations and makes them easier because nothing is independent of time. However, it is important to keep in mind that in most systems it is not time that changes the multi-directional endogenous dynamics but changes in other system characteristics (Word processor example) account for that. These changes can happen faster or slower at certain times. Hence, the path-dependent processes discussed below are in most cases not simply triggered by time but by technological or cultural developments.

3.4 Potential function

The potential function of the Fokker–Planck equation is the central tool in our discussion in the remainder of this paper. Therefore, it is necessary to understand its

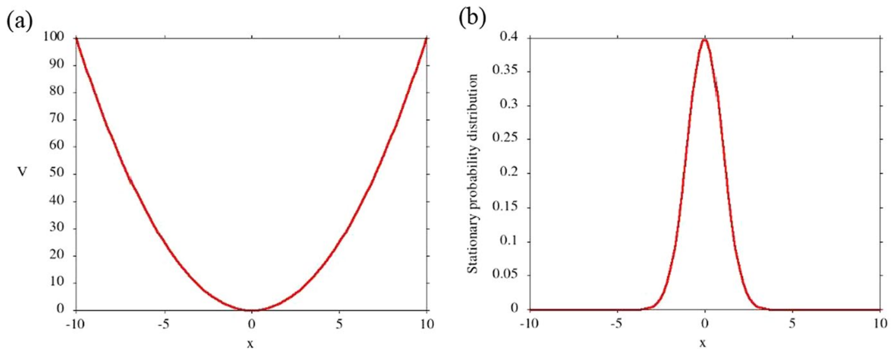


Fig. 1 Exemplary potential function (**a**) and stationary distribution (**b**)

meaning. Mathematically this potential function is defined by the following characteristic: its slope represents the deterministic dynamics of the system divided by the strength of randomness (see Appendix). This helps our depiction because the potential function can be interpreted as a landscape and the dynamics of the system are then well described by the imagined downhill movement of a ball in this landscape.[1] A similar metaphor to this has already been formulated by Arthur (1988).

An example of a potential function is given in Fig. 1a. In this case the ball would roll down to the middle, independent from where it started or if random processes were involved. The same would happen in case of a system without random processes. However, the potential function only represents the causal dynamics of the system. As argued above, real-world systems always contain randomness. This means that we must add random movements to the ball's dynamics in the landscape, causing it to move, on average, downwards but ending up dithering around the bottom of the valley. The stationary distribution (Fig. 1b) eludes this behavior clearly: A stationary distribution provides the likelihood with which the ball is found at each position if we look at it at any time in the long run, considering randomness. In our case it shows that the ball is most likely found in the middle (depicted by the highest value at 0 in Fig. 1b) but also still quite likely somewhat left or right of this, while it is more and more unlikely to find the ball far away from the bottom of the valley.

In the following it is important to keep in mind that the potential function represents only one part of the dynamics of the system: the causal dynamics. The movement of an imagined ball in the landscape of the potential function represents the most likely development of the system. A second example of a possible potential function (a) and the respective stationary distribution (b) is depicted in Fig. 2.

This case is comparable to the systems analyzed in some of the early works on path dependence (e.g., Arthur et al. 1983, 1986; Arthur 1988, 1989). Depending on

[1] This is not entirely correct because a ball (ignoring air resistance) would have the highest speed when reaching the bottom of a valley, while our system has the highest speed at the place with the steepest slope. However, most people intuitively assign the highest speed to the steepest slope, so that they will interpret the graphs correctly.

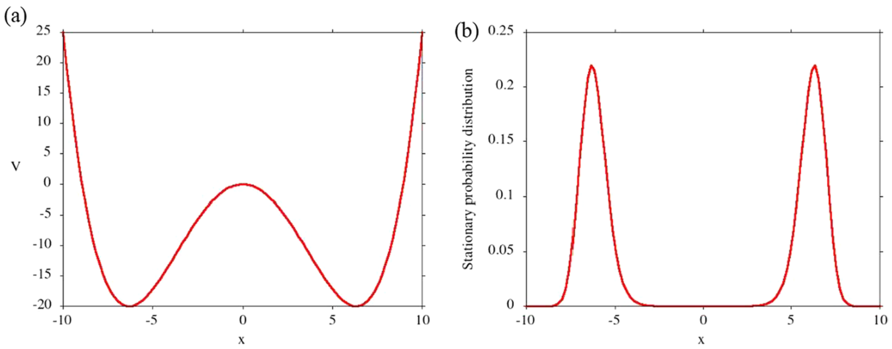


Fig. 2 Exemplary potential function (**a**) and stationary distribution (**b**)

the current state, the system tends to develop toward the nearer valley of the two. Nevertheless, due to random processes, systems starting in the middle might move toward any of the two valleys. However, once the system has moved to the ground of one valley, it is very unlikely that it leaves this valley again without an external status change. Figure 2b shows that the probability to climb up the hill in the middle of Fig. 2a is very small. Assuming that the ball is in one valley, the system then reaches a lock-in and the attained state (valley) depends predominantly on the initial random processes and is therefore, highly path-dependent. The strength of the lock-in depends on the height of the hill in the middle (see also the Appendix). These examples show that when considering only one dimension, the above model reproduces the familiar findings on path dependence and lock-in.

Due to limitations in drawing multi-dimensional graphs we present illustrations only for systems with one multi-directional variable below. But certainly, this raises the question of how systems with more than one multi-directional variable can be treated.

As mentioned above, many path-dependent systems consist of a population of agents that have different options. For plausible illustration let us consider such a system with three options for the agents. The state of the system is then given by two variables representing the shares of the agents that follow option 1 and option 2 (the share for the third option can be calculated from these two). Hence, there are two multi-directional variables describing the system and for simpler assumptions about the behavior of the agents we include an incentive to follow the choice of other agents. Following this, we obtain the potential function depicted in Fig. 3a. Similar to the situation in Fig. 2, there are valleys that the system will move to, and it will be unlikely for the system to leave a valley once it moved there. Hence, the basic characteristics of given valleys is the same as above, just with three instead of two valleys. This situation can also be represented in a two-dimensional graph (Fig. 3b). However, this reduction of the representation loses two aspects: First, there are multiple paths from one valley to another in the original graph (Fig. 3a), while there is only one in the simplified graph (Fig. 3b). Second, there is an order in the simplified graph implying that there is no direct path from the left to the right valley without passing the valley in the middle in Fig. 3b. This does not accurately correspond

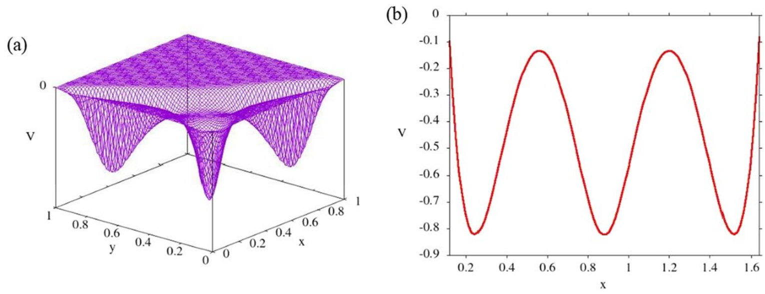


Fig. 3 Exemplary potential function on a two-dimensional state space (**a**) (in a three-dimensional coordination system) and one-dimensional abstraction (**b**)

to the situation in the original graph in Fig. 3a. Although impairing the accuracy, both losses in precision do not matter for our later discussion of path-dependent processes. All definitions of these processes are based on the existence of the valleys and the probabilities to switch between them, no matter how many different ways of these switches could exist or whether they are directly or indirectly possible. Hence, our classification and discussion below does not depend on the exact number of multi-directional variables. Nevertheless, we should keep in mind that the multi-directional axis stands in most of the real cases for many dimensions and that the valleys depicted in a sequence might be in a more complex multi-dimensional space.

3.5 Two-dimensional developments

As stated above, we argue that it is important to consider at least two dimensions: one representing a multi-directional variable and one representing a uni-directional variable. In the following we consider exactly these two dimensions because more dimensions would not hold for a graphical representation of potential functions. This implies that many real-world systems cannot be represented exactly by the following graphs (as also discussed in the subsection before). Nevertheless, all basic processes can be shown and discussed. Applying the model to all kinds of real-world systems requires to imagine the graphs in a more than three-dimensional version. This is possible but cannot be depicted understandably. We will give some hints on this in the discussion of the processes.

The value of the uni-directional variable is, on average, always increasing with time. This implies that the respective potential function is down-sloping. The potential function can be linear, meaning that the dynamics for the uni-directional variable are the same for all times, or can show more complex structures, as depicted in Fig. 4. More complex structures imply that there are times of faster and slower average developments in the uni-directional variable. For most of the following discussion the speed of the dynamics of this variable is not decisive. Consequently, we use a linear potential function as the standard case to keep it simple (the first graph in Fig. 4).

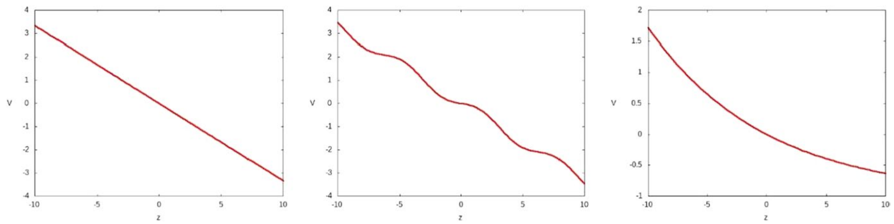


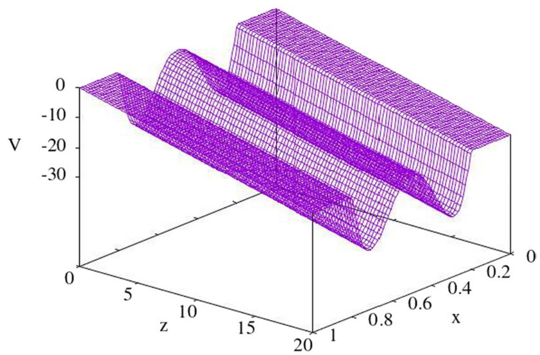
Fig. 4 Exemplary potential functions for the dynamics of the uni-directional variable

Above we have discussed the potential functions for the two dimensions separately. However, a joint potential function is required. Mathematically such a two-dimensional potential function only exists under specific conditions that are rarely satisfied outside of theory. Nevertheless, we first discuss such a specific case to provide a deeper understanding before we present the more general case.

The simplest case in which a two-dimensional potential function exists is when both dimensions are independent of each other. A good example is the keyboard layout (see above). We assume that there are two keyboard layouts (the use of them represented by the x-dimension and the potential depicted in Fig. 2a). At the same time, there is a technological development (represented by the z-dimension and Fig. 4) from typewriters toward computer keyboards and whatever comes next. However, the use of keyboard layouts so far does not seem to be influenced by technological developments. Therefore, the two dimensions are independent, and we can simply merge the potential functions as depicted in Fig. 5. The keyboard layouts are represented by the x-axis with the two clear valleys, whereas the technical development is represented by the z-axis showing a clear down-sloping. The graph shows that once one of the keyboard layouts dominates (reaching one of the valley grounds), the technological development does not interfere with the stability of this choice and the most likely development follows the valley ground.

In such a specific case, no new information is gathered by introducing the second dimension. There is still a lock-in into one of the valleys and the system simply

Fig. 5 Exemplary two-dimensional potential function



develops within the valley that is reached due to random processes occurring in the beginning. This is comparable to a ball rolling down in one of the two grooves.

The more general case is the one in which the two dimensions depend on each other, implying that the potential function in one dimension changes because of a change of the state in the other dimension. In our example, this would mean that the technological development changes the attractiveness of the various keyboard layouts and/or the probability to use them. The problem is that in the case of connected dimensions, a joint potential function does not exist. This is even more the case as we must keep in mind that reality cannot be represented adequately by two dimensions.

However, since the mathematical model is only used as background for building a metaphor to understand path-dependent dynamics, we do not depend on the existence of a potential function for the whole system. We use the following: For each value of the uni-directional variable (z) the potential function for the multi-directional variable (x) can be calculated. The uni-directional variable develops, except of rare random events, only in one direction. Hence, over time the system usually moves through the values of z in one direction. And for each value of z a potential function can be calculated for the multi-directional variable (x). We put together those potential functions to obtain the potential function landscape.

As a consequence, the depicted potential functions are correct only for certain paths of developments. If a different path would be taken by the system, the potential would look different. The difference could be large if there is more than one multi-directional variable, which is likely in reality. Hence, the potential functions that we draw in the following represent the likelihood of deviations and alternative developments for a given path (e.g., the historically observed path). The possibility that the potential function could look different if another path is taken increases the more the multi-directional variables are interconnected. Especially in the case of path-furrowing (Sect. 4.4), this becomes relevant.

4 Classes of path-dependent processes

The aim of this paper is to classify the different path-dependent processes and by this establish a common denominator. The crucial assumptions underlying this approach are summarized here again to provide an overview before analyzing the different definitions:

- The system is assumed to be characterized by uni-directional variables (represented by one z-dimension) and multi-directional variables (represented by one x-dimension).
- The likelihood of potential developments can be depicted by a potential function: (steep) uphill represents (very) low probability, and (steep) downhill represents (very) high probability of the respective direction of developments.

- Randomness is a crucial element, especially resulting from the individual behavior of actors, which might deviate from what can be predicted by common incentives and characteristics.

While discussing each kind of the path-dependent processes we also discuss matching definitions in the literature and use the respective terminology if possible. For a less abstract view on the following illustrations, the reader can interpret the multi-directional variable (x) as share of one behavior or choice in the population, e.g., the share of QWERTY keyboards, and the uni-directional variable (z) as time. However, the use of the graphs is much more general, so that we abstain from using explicit labels.

4.1 Paths and path dependence

As mentioned above, a situation most in line with the original mathematical modeling by Arthur (1983, 1989) is depicted in Fig. 5. Still, since our understanding includes not only one but two dimensions, the system will not converge to one exact state and stay in this state forever. Random fluctuations (for example other keyboard layouts such as Dvorak) and the continuous development in the z-dimension cause the dynamics we are specifically analyzing.

Consequently, our new understanding fits the wording used in the path dependence literature even better. Thus, while originally the path was seen as the development toward a stable state, we define:

Definition 1a A *realized path of a system* is the real development of the system in time.

There is always one realized path, which can be observed but not predicted ex-ante due to random events.

Definition 1b A *maximum-likelihood (ML) path of a system* is the most likely development of the system in time from a given initial state.

In the graphical representation, the most likely development is the development along the bottom of one of the valleys of the potential function. The system described by the potential function in Fig. 5 has, after some initial dynamics (convergence into one valley), two maximum-likelihood (ML) paths that are represented by the two valleys in the graph. Given the existence of random events, the system will not precisely follow the bottom of the valley like a river would do but it will most likely stroll around its most probable development. Hence, the *realized path (“strolling path”)* will not be identical to the *ML path (“the river”)*. However, due to the uphill slopes on both sides of the valley, representing small probabilities for further developments away from the valley and thus large probabilities for moving back to the valley bottom, the system will stay in the valley and near the most likely

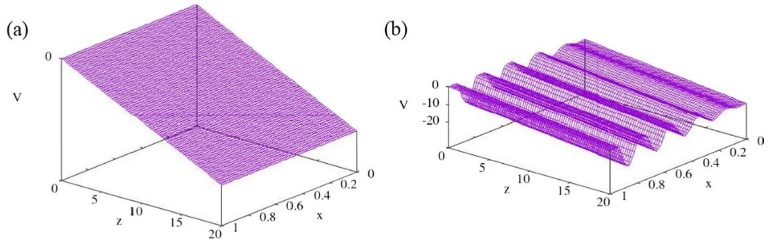


Fig. 6 Potential functions for (**a**) a system without ML paths and (**b**) a system with several ML paths

development (ML path). This results in a realized path that is usually not far away from the ML path.

According to the above definition, Fig. 6a shows a case in which no ML path exists. Besides the fact that there is no most likely development, forces that bring the system back to a given path are missing. To satisfy Definition 1b, valleys in the potential function are necessary. However, the definition does not depend on the depth of the valley. All exemplary valleys with different depths in Fig. 6b constitute a ML path.

Nevertheless, the example depicted in Fig. 6b highlights a problem in the definition of path dependence for random systems. Due to random events, the system might move from one valley to another, leading to switches between different ML paths. This might be very unlikely, as in the case of the more sinistral valleys in Fig. 6b, or very likely, as in the case of the more right-handed grooves in the same graph. However, as previously mentioned, most systems contain aspects that develop (nearly) irreversible (*z*-dimension) so that they are path-dependent according to the usual definitions anyhow.

But this is not what most applications are interested in. Conversely, research objects are usually systems that have two or more alternative possible ML paths. Therefore, we define path dependence using the above definition of ML paths and extend it to:

Definition 2 A system is *path-dependent* if two or more maximum-likelihood (ML) paths exist for the considered period.

The above definition of ML paths clarifies that being on one path makes continuing this path the most likely further development. However, this definition does not address whether leaving a path is likely or very unlikely. We come back to this issue in the next subsection.

The additional phrase "for the considered period" is used intentionally because paths might end due to the development in the *z*-dimension. Examples are given in Fig. 7: developments in the z-dimension might cause (a) the forces constituting the ML path(s) to disappear, (b) one ML path to vanish, so that only one ML path remains, or (c) the paths to converge to each other so that finally only one ML path remains. Hence, path dependence can be a temporary phenomenon. Usually, we can determine at least roughly when it started (see Sect. 4.6 below) but the future is

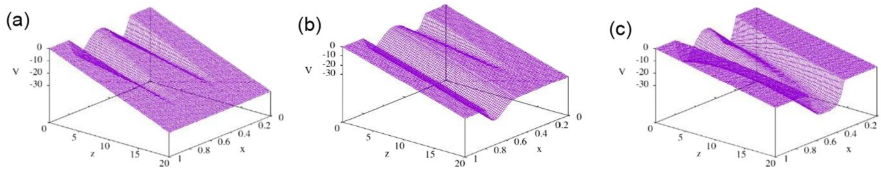


Fig. 7 Potential functions for systems with (**a**)/(**b**) disappearing and (**c**) blending ML paths

difficult to predict. Changes in the functioning of the system that could lead to the disappearance of the actual ML path(s) are always possible.

Applying this definition of paths and path dependence requires considering the following aspects:

- *Defining the system*: What belongs to the considered system? What are the levels that might influence the system (considered as primary causes for random events)? Which are the actors whose decisions are the second source of randomness and therefore could (predictably) influence the system's development?
- *Identifying the ML paths*: Which developments take place independent of the realized path (z-dimension)? Which are the options for the development of the system?
- *Identifying the self-reinforcing mechanisms*: What causes the system to return to the ML path(s) after small deviations and therefore create valleys in the potential function?

These aspects meet the constitutive features (the level analysis) declared by Sydow et al. (2012, p. 159): The system definition corresponds to "level interrelatedness" and "multiple actors". The identification of ML paths links to their "non-ergodic process", which is neither a determined nor an arbitrary path. However, we do not think that the mathematical characteristic of non-ergodicity fits the processes that are usually addressed in the context of path dependence (see discussion above). Another accordance can be found in self-reinforcing mechanisms, corresponding to their "self-reinforcing processes" (Sydow et al. 2012, p. 159). Triggering events and lock-ins are of specific scientific interest in the context of path dependence but are not required basic features of path dependence per se. In the following subsections we will discuss the relating concepts of path dependence and connect them to our graphs as well as to definitions in the literature.

4.2 Lock-in

If a system is path-dependent in the above-defined form, suggesting that there are two or more ML paths, it might change from one path to the other due to random events. The usual definitions based on non-ergodicity imply that a change between ML paths is not possible. We believe that moving to another ML path is always possible in real systems but usually it is very unlikely. Here again, the keyboard layout is a good example: Alternative layouts do exist, and some people use them. In our opinion, if, for some reason, more than half of the population would switch to the same other keyboard layout at nearly the same time, we would get to another ML path. This is very unlikely, though, but there is a probability for such a switch. This probability could even be calculated using the actual number of users of alternative layouts. The resulting probability will be extremely low but not zero.

Thus, in line with our mathematical modeling, we assume that moving to another ML path is always possible. The likelihood of such an event is the crucial issue. If the likelihood is very high, ML paths are not very stable. We deliberately decided above that we call those systems path-dependent, independent of the likelihoods for path changes. Even if alterations of ML paths are likely, the actual ML path the system follows has an impact on further developments because it is the most likely path for the future. This situation could also be defined differently, when only systems with low probabilities for ML path alteration are labeled path-dependent. However, a threshold for the likelihood of path changes is difficult, not to say impossible, to define and to apply in real case scenarios. Therefore, we define path dependence very broadly and use the terms “lock-in” and “path-breaking” for those cases that are difficult to escape, following the argumentation of the literature (e.g., Vergne and Durand 2010).

Looking deeper into these “stronger” cases of path dependence, we have to analyze the source of path changes. Above, we defined three reasons for randomness: random actions, intentionally/organized deviating actions, and random change of external circumstances. Bassanini and Dosi (2001) distinguish between big events which occur rarely and small events which occur frequently. This distinction is helpful and adopted here. *Random actions* refer to differences in the behavior of actors due to differing preferences, unconsciousness, or simply mistakes. In a system with many actors such randomness is always given but usually of minor impact. *Intentionally and organized deviating action* is understood here as those actions that intend to change the system. Consequently, these actions are rare but can have a strong impact. Finally, *changes in external circumstances* can be both, frequent and small as well as rare and large changes. Since the complete range from frequent, small to rare, large events is possible, they are difficult to classify a priori. Therefore, we focus on the actors in the system in the following discussion.

We argue that if the usual variety in the behavior of the actors can move the system from one ML path to another, the system’s path dependence is weak. In the case of the keyboard layout, this would indicate that the people deviating from the QWERTY layout might occasionally become such a large part of the population that the standard layout is changed, which we do not expect to happen. In contrast, if the small events caused by the variety in behavior are not sufficient to get to another ML path, the system is strongly path-dependent. In this case, only intentional and organized deviating actions or large exogenous events might lead to path changes. In the case of QWERTY, this would represent an initiative to change the keyboard layout that succeeds in getting policy support and finally leads to a change of the layout by governmental rule. We therefore differentiate weaker and stronger path dependence and consequently develop the following definition of a lock-in:

Definition 3 A *lock-in* is given if a system follows a ML path and, at least for a certain period, another ML path exists simultaneously but the usual variations of actor's behavior have a very low probability (are in principle unable) to lead to a change to this other ML path.

This definition allows identifying lock-in situations in applications by examining whether intentional, organized action or large external shocks are necessary for changing the current path. A mathematically correct definition would require determining a threshold for the term "very low probability". We could now discuss such a threshold but in real systems an exact probability for such a ML path change cannot be calculated.

In the literature various formulations and definitions of lock-ins can be found that somehow range from "a temporary stabilization of paths in-the-making" (Garud et al. 2010, p. 760) to "a hard-to-escape situation" (Vergne and Durand 2010, p. 743). Our definition is nearest to the definition by Martin and Sunley (2010), who distinguish two types of lock-ins: in the first, one of many possible paths becomes locked in until at some point an external shock dissolves the path. The second type is when a technology or industry becomes temporarily locked in until there is a better technology or industry which then replaces the previous equilibrium-lock-in state. Both types are included in our definition because due to the development in the uni-directional variable (z-dimension) ML paths might dissolve, merge, or appear (see below).

4.3 Path-breaking

We use the aforementioned understanding also for the following definition:

Definition 4 *Path-breaking* is the intentional, organized action of one or more actors that leads to a change from a locked in ML path to another ML path.

Hence, path-breaking is the dissolution of a lock-in situation intentionally organized by actors, excluding external events or developments in the uni-directional variable z. The definition of path-breaking crucially depends on the definition of the system and the processes considered as endogenous. Usually, the endogenous mechanisms represent the common incentives, interactions, and behaviors of the agents of the system. In addition, the random processes in the model reflect the individuality of the agents, leading to deviations from the average action. The QWERTY keyboard is again a good example. While there is some deviation from using this keyboard layout, a model based on the considerations of users, including availability, teaching, and compatibility, predicts an extremely low probability for leaving this locked in ML path. Hence, endogenous processes will not break the path. There are three options for leaving a locked in ML path: (1) There might be an external shock that completely changes the processes and characteristics of the system, e.g., typewriting does not exist anymore. We do not call this path-breaking because the model would have to change as well as all ML paths. (2) There might be technological or social developments over time that change the probabilities to leave a ML path (this is classified as a different process below). (3) The agents of the system might interact in such a way that they are able to break the lock-in, for example by setting up a law to change all keyboards. In principle, such an action could be included in the model, which would make it an endogenous process. However, building an understanding of a system based on the incentives and individual actions of the agents therein is much easier than predicting the occurrence of joint action beyond that. Hence, it seems adequate to differentiate such intentional and organized action from an expectable development along a ML path.

The parallelism to Garud's and Karnøe's path creation (2001) sticks out at this point, respecting the intentions of actors. However, in our case the aim is not to create a new path but to dissolve the former one for the sake of reaching a different, already existing ML path. Therefore, our definition of path-breaking is a subsample of the path creation process described by Garud and Karnøe (2001).

4.4 Path-furrowing

Our approach leads to a valuable distinction that has so far not been discussed explicitly in the literature, although some existing arguments fit our definition. We hypothesize that the potential function depends on the actual state of the system. This implies that the potential function and therefore also the ML paths and their shape change, depending on the realized path. The difference reveals itself in its details: To explain them, we compare the situation depicted in Fig. 5 (the exemplary two-dimensional function) with a process in which the shape of the potential function changes.

In the case of a fixed potential function given by Fig. 5, the two ML paths are both stable, implying that it is unlikely that the system leaves the ML path that it develops along. However, if the system changes to the other ML path for any reason, this path becomes immediately stable again.

Nevertheless, this is not the only possible outcome. It is also possible that the potential develops as depicted in Fig. 8a implying that leaving the actual path, here the right one, becomes more unlikely with time. From this perspective, the other ML path (left side) is far more likely to be left. However, if the system changes to the other path during the depicted development, the potential function can change as well, e.g., to the form in Fig. 8b. In this case, the probability of leaving the "new" ML path again, directly after the change, is high but decreases with time. This means, that the stability of a ML path develops over time and is not given initially. In real-life scenarios, we often see such processes caused by the development of complementary aspects: Possible examples could be societies concentrating on a certain technology and the related, maybe specialized, economic developments, or psychological aspects, such as habit formation. Therefore, we propose the following definition of path-furrowing:

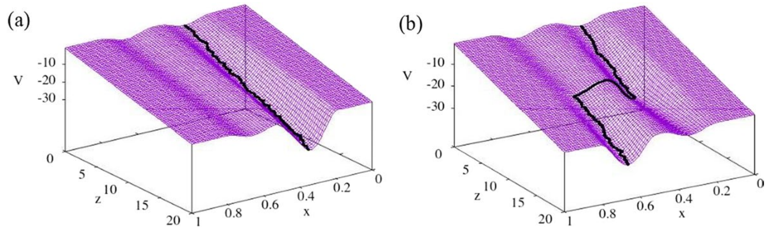


Fig. 8 Exemplary potential function that changes depending on whether the system develops along with one (**a**) or suddenly along with the other (**b**) ML path (black line: exemplary realized path)

Definition 5 *Path-furrowing* takes place if the likelihood to leave the ML path decreases with the time following this path.

The situation of path-furrowing can be clearly distinguished from a situation in which the state of the system completely determines path dependence. With path-furrowing, the likelihood to leave a ML path depends strongly on how long the system has already developed along this path. We can determine which kind of path dependence is present by looking at the self-reinforcing processes: If the self-reinforcing processes need time to develop, path-furrowing is present, while already present self-reinforcing processes lead to path dependence without path-furrowing. In applications, we often see both. To clarify, we can resort to another famous path dependence example: The competition between the two video systems VHS and Betamax. The two companies Sony and JVC both brought video recording mediums to market in the 1970s that were not only able to store videos but also made them exchangeable. The cassette by Sony was called Betamax. It was obtained as the superior system and was distinguished by a smaller size, whereas JVC with its brand VHS produced larger cassettes and scored with a longer recording time[2] (Puffert 2019, an exemplified rivalry early described by Liebowitz and Margolis 1995, see also David 1997). The more people use one of these systems, the more attractive the system becomes for further users due to network externalities, e.g., the possibility to exchange videos and the availability of equipment. This is a self-reinforcing process that occurs immediately depending on the number of users. Interestingly, the situation is completely different for the use of different car engines. Let us assume that all people use a combustion engine. If for some reason 90% of the population would change to an electric engine at one day, this would not make the choice of an electric engine more attractive, because the conditions do not change equally fast. Thus,

[2] Those readers born after 1995 might not be familiar with the video recording systems of former times. Video cassettes and the incidental machines were the first affordable possibilities to record and store videos at home. Betamax was the technically advanced system with a better picture quality, a smaller cassette size and some other benefits. VHS cassettes were bigger but had the advantage of a longer recording time which was especially interesting for the customers in the USA who wanted to record the American football games. Nevertheless, the recording time was not the only factor that made VHS the leading brand but also their strategy to enter the market and gain suppliers as well as consumers played an important role (Puffert 2019).

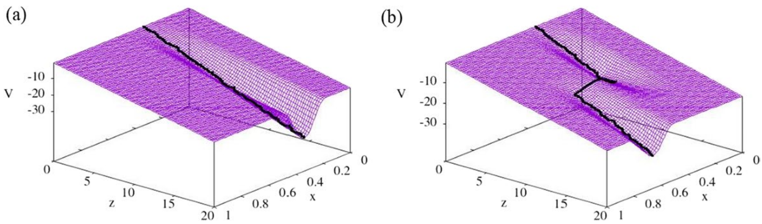


Fig. 9 Exemplary potential functions for systems with path-furrowing (black line: exemplary realized path)

the corresponding infrastructure would still be missing, and people would have an incentive to move back to the combustion engine. However, if some people continue to use electric engines, e.g., due to subsidies, there is an economic incentive for other actors to establish the respective infrastructure. This takes time but continuously the electric engines become more attractive and finally a stable new ML path is built. It shows that path-furrowing intensifies with time. Usually, two related aspects with at least one of them reacting slower than the other are responsible for this kind of process. It is important to mind the difference to the development of a system toward a given ML path (a developing valley in contrast to an existing valley in the above illustrations). Of course, the further the system has converged to the valley, the less likely it will depart from this ML path. The difference is that a strong exogenous event (e.g., the government subsidizing the use of the Betamax video system), that moves the system to another ML path, could immediately make this ML path stable. Usually, this then results in a lock-in due to the immediate benefits from showing the same behavior as the majority. Path-furrowing does not show this mechanism. In the case of the electric engine for example, exogenous influence is necessary until the respective infrastructure is established and the new path really becomes a “stable” ML path. Following this, change in the case of path-furrowing is much more difficult. This important difference is, to our knowledge, not highlighted in the literature.

Path-furrowing can also lead to the situations depicted in Fig. 9. In these cases, no ML paths exist a priori. Instead, they establish while the system develops along with them. Once a path is taken, all others stay invisible (Fig. 9a). If the system leaves a ML path in the middle, this path will slowly disappear while another path that is taken becomes the ML path (Fig. 9b). Like in all other cases but here especially, the ML path can often only be identified ex-post in real-life scenarios.

4.5 Path plasticity

So far, we have considered randomness only in the context of change between different ML paths. However, randomness also has an influence if the system follows one ML path. In this context, we can distinguish four developments:

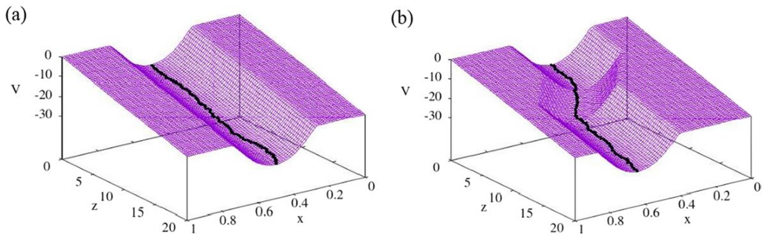


Fig. 10 Slight change of the potential function and path due to path-furrowing on the valley bottom (**b**) in comparison to development without path-furrowing (**a**) (black line: exemplary realized path)

(1) *Wide-range development*: Systems do not exactly follow the ML path. Randomness causes some fluctuations along this ML path. However, the aforementioned definition of a ML path includes the system's tendency to return to the path after minor deviations. This indicates that the system strolls around the ML path, giving space for situations in which the valley of the potential function is wide so that the fluctuations themselves can become larger (see e.g., Fig. 10a).

(2) *Random development velocity*: Randomness does not only affect developments in the x-dimension but also in the z-dimension. So far, we have discussed the developments as if the dynamics in the z-dimension are continuous and uniform. However, this does not have to be the case. On the one hand, the potential function must not be linearly decreasing in the z-direction. On the other hand, random events lead to faster and slower developments at different points.

(3) *Path-dependent development velocity*: The slope in z-direction depends on the exact path that is taken. This means that deviations in x-direction due to random fluctuations can also influence the speed of the development in the z-direction. However, as in the case before, this does not change the path but solely changes the speed of developments.

(4) *Path-furrowing in path plasticity*: In addition to pure randomness, path-furrowing can be important to path plasticity. The system does not develop exactly along the bottom of the valley but strolls around it due to small random events (see Fig. 10a). If path-furrowing takes place, the shape of the valley might slightly change. Since path-furrowing takes time, the tendency of returning to the given ML path remains and thus still fulfills the requirement in the ML path definition. Nevertheless, if the system remains on one side of the valley bottom for some time by occasion, the valley bottom might move in this direction and the path is slightly but steadily changed (as shown in Fig. 10b in comparison to Fig. 10a).

All mechanisms described above lead to deviations of the realized path from the ML path while they do not change (1–3) or only slightly change (4) the ML path. The first three mechanisms mentioned here highlight that ML paths are not followed exactly, which does not further impact the direction of the ML path. The fourth mechanism is different because the ML path changes. However, this movement is rather at a small scale, as long as no structurally relevant changes occur. Nevertheless, larger changes are possible. However, we focus on changes of the ML paths that are based on random events, more likely small events, and the subsequent path-furrowing processes. The path-furrowing processes are adaptations of the system to the events that consolidate the change in the path. This is in line with the definition of path plasticity by Strambach (2010, p. 407, see also Strambach and Storz 2008, p. 148), who states that path plasticity "describes a broad range of possibilities for the creation of innovation within a dominant path of innovation systems". In the context of our modeling, we define:

Definition 6 *Path plasticity* takes place if small deviations, caused by random events, from the ML path are consolidated by adaptations of the system (path-furrowing) so that the ML path is slightly changed without changing its basic structure.

Path plasticity was initially introduced to explain (incremental) institutional innovations. Strambach and Halkier (2013, p. 11) highlight its characteristic as follows: "The good news here is of course that in contrast to a lock-in/path dependency perspective on regional economic development, change will not have to be brought about through external shocks but may be the result of incremental adjustments and variations".

To distinguish the processes, we exclude in the above definition all structural changes of the ML path, including changes that cause ML paths to disappear (see Sect. 4.1) or new ML paths to appear (see Sect. 4.6). In addition, our approach makes clear that path plasticity requires the presence of path-furrowing.

4.6 Path formation

Previously we argued that ML paths are a temporary phenomenon. In Sect. 4.1 we showed (Fig. 7) that ML paths can disappear. Similarly, ML paths can emerge. One example is given in Sect. 4.4 where systems with no initial ML paths can develop such paths by path-furrowing (e.g., Fig. 9a). Besides this, a ML path can appear due to developments in the z-dimension (Fig. 11a). More relevant cases are those, in which two or more paths appear at a certain point in time (e.g., Fig. 11b, c), and cases in which one path splits into two or more paths (e.g., Fig. 11d).

All these cases can be subsumed under the following definition:

Definition 7 *Path formation* takes place if the number of ML paths of a system increases due to the development in the uni-directional variables (z-dimension).

In the following, we distinguish and discuss five types of path formation based on the potential dynamics that result from it:

(1) *Emerging path formation*: In a system without ML paths, the occurrence of path-furrowing generates a situation with an emerging ML path (see Fig. 9a). The relevant aspect in a situation like this is that there are many options for the characteristics of the emerging path since wherever the system moves to is self-

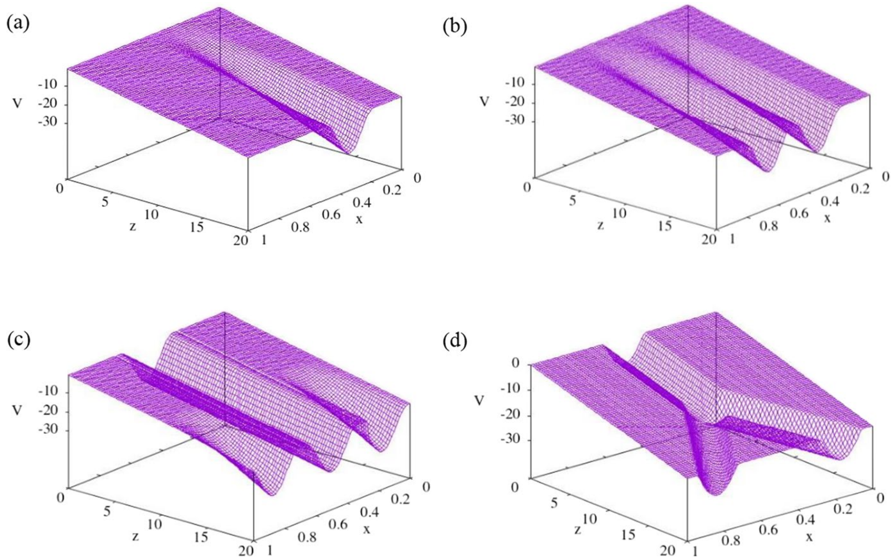


Fig. 11 Different cases of potential functions with path appearance and path duplication

reinforced by the furrowing process. Hence, there are multiple options for the further development of the system.

(2) *Predetermined path formation*: In a system without ML paths, one ML path might appear. The respective potential function (Fig. 11a) is identical to the one in Fig. 9a, but the mechanisms are completely different from those in the case before. The valley in the potential function in Fig. 11, defining the ML path, is determined by the system's characteristics and mechanisms. Hence, the valley is determined to exist at this place right from the beginning. It only requires time for it to become relevant. After this, and in contrast to emerging path formation, there are no other options than following this predetermined ML path.

(3) *Predetermined multiple path formation*: In a system without ML paths, two or more ML paths might appear at a certain point (e.g., Fig. 11b). In this case, different options for the further development of the system appear. It will at least follow one of the ML paths. The initial state determines the likelihood of each path to be taken. The likelihoods are not definite, meaning that small random events decide which of the possible ML paths will be followed.

(4) *Determined further path formation*: In a system that follows a ML path, further ML paths can appear without a dissolution of the current path, and they coexist in the following. However, the system has a low probability to reach the other paths (see e.g., Fig. 11c). These further ML paths only become relevant for the development of the system if the current ML path is exited, e.g., due to path-breaking.

(5) *Split up path formation*: In a system that follows a ML path, the path might split into two or more ML paths (see e.g., Fig. 11d). The exact shape of the splitting might make one ML path more likely than the other(s), but the further devel-

opment of the system is influenced by predominantly small random events. A similar case is the disappearance of the ML path and the appearance of two or more new paths. Concerning the dynamics of the system, this case does not differ from the case of a splitting ML path, so that we do not distinguish them. David (2007, p. 100) described this situation by: "What is meant here […] is simply conveyed by the metaphor of a wanderer arriving at a 'fork in the road', from which diverging trails lead to two or more distinct regions between which there are no other connecting routes".

Let us again use the video recording systems to demonstrate the processes 2–5 in a real example: Any new developed video system would have immediately led to the occurrence of a ML path (a valley in the illustration) defined by all or most people using this system. Hence, the appearance of the ML paths is a consequence of the technological development (in the z-dimension). It is clear from the structure of our economy and society that once video systems become technological possible, each developed video system forms an ML path. Thus, while the video systems themselves are intentionally created, the structure of the path-dependent system that we describe is a consequence of technological development.

However, we face a question of perspectives: We argue that all these ways of path formation are not done by deliberate actions but need to be considered as features of the system and the mechanisms and processes involved. The system has been defined above to include all possible decisions of actors, even the unlikely ones, and the technological development as uni-directional variable. Hence, all specific actions are also included and cannot generate something new to the model. This is a theoretical perspective and depends on the modeling. In real-life scenarios, people generate all technological and social progress. However, if we interpret this as exogenous to the system and not as a character of the system, then there are no ML paths. To give it a visualization, the question is whether we assume that humankind would have developed a wheel at some time anyhow. One perspective is that the advantages of this tool are so strong that it was only the question of who and when it is developed. This means that the path was given but the speed was determined by actors represented in the model as randomness. The other perspective is that the invention of the wheel was an unexpected act opening a new path. The fundamental question here is what unexpected actions are. If we argue that human behavior cannot be modeled, all actions would each generate a new path because it would be unexpected and has an influence on the future. We, instead, rather argue that a complete model includes all possible behaviors, even the unexpected ones, meaning the events with extremely low probability. Nevertheless, such a model is a theoretical stylized conception. Still, it is the basis of theoretical concepts as the one we developed here.

4.7 Path selection

Nevertheless, a theoretical perspective like this does allow for deliberate action influencing the path that is realized. In three of the above-described cases of path formation, cases 1, 3 and 5, the system has several potential paths it could follow in future at a certain point in time. According to the model, it depends on random events which path is followed. However, random events are representations of external events and decisions of the involved actors. Therefore, actors can influence or even choose the path at specific occasions. “Extreme” actions, meaning actions with a low probability, can conduct the system to paths that seemed very unlikely. Thus, there are from time-to-time so-called windows of opportunities (Öberg and Hallberg Adu 2009; Magnusson and Ottosson 2009) in which the actors become decisive and strongly influence future paths. This coincides partly with the understanding of path constitution, path generation or path creation in the literature. However, these various concepts differ slightly and do not exactly fit the process we have in mind here, so that we define:

Definition 8 *Path selection* occurs if several ML paths exist in the system at a certain point in time and intentional actions determine the subsequent ML path.

According to our understanding, the path is not created nor generated by the actions but already exists due to the mechanisms and processes in the system and is then “selected”. This is intentional and not the sum of many random decisions. The latter one is the alternative process if no actor is aware of the different options or willing or able to choose or influence the developments intentionally. In such a case the possible ML paths are chosen with the probability that is assigned to them by the model.

Our definition of path selection deviates most from the concept of path creation in the literature. While we are in line with the focus on the subjection of human actions that remains implicit in path dependence (Garud and Karnøe 2001), we do not confirm the idea that path creation is based on mindful deviation. We see it rather as a mindful choice of options. In our approach, mindful deviation would rather fall into the concept of path-breaking. The various definitions of path constitution, path generation or path creation presently range somewhere between path selection and path-breaking. All those processes have in common that actors intentionally influence the further development of the system based on the existence of various ML paths, which either emerge at that time or already exist.

5 Conclusions

The paper at hand develops a graphical concept to present and discuss path-dependent processes and classify and characterize the different types. This allows us to define the various path-dependent processes within one theoretical framework, clarifying their differences and setting a common denominator for further path dependence application. A central innovation to the discourse about path dependence is the division between the realized path and the maximum-likelihood (ML) path. Whereas the first is the path that is actually followed, the second path is the one with the highest probability to be followed.

Our model explicitly goes beyond the original models by Brian W. Arthur and Paul A. David. While the initial models describe systems that are fixed by a given set of possible states, we model systems with evolving state spaces, which have so far only been slightly investigated. The main steps and richness of broadening the perspective lie in the inclusion of human behavior as random events and the distinction between uni-directional and multi-directional variables. This is important to involve the different levels that influence path dependence: We argue that systems in reality are in most cases characterized by evolving state spaces, caused by developments that are leading in one direction, e.g., technological developments.

Within our framework we define the processes of lock-in, path-breaking, path plasticity, and path formation. Since these processes have been already defined and discussed, we provide the scientific community with clearer and more explicit definitions of these processes, including a precise distinction between them. This will help scholars of any scientific direction by providing a distinct tool box for classifying and categorizing all existing path-dependent processes precisely. In addition, the approach led us to detect the process of path-furrowing, which has not been explicitly identified in the literature so far. Path-furrowing gives credit to the interaction between variables generating a gradual emergence of path dependence, it supports the importance of slight changes within the ML path and pays attention to steadily solidifying path dependence. Additionally, we go beyond the known set of mindful actions within path-dependent processes and introduce the mindful path selection. This allows for a clearer distinction between the path formation due to system characteristics and dynamics, and the mindful action of selection and the possibility of choice among existing paths.

This paper and the distinctive definitions encourage further research to check for the various processes in real developments. The diverse and vivid depictions unite not only a common understanding of the different processes but also the possibility of application to manifold research situations. This last step is not proceeded here because it goes beyond the scope of this already extensive paper. However, we hope that many researchers will apply the provided definitions to numerous developing systems.

Appendix

Mathematical model and potential function

The following model builds the background of our approach and is used to understand the potential dynamics of the herein analyzed systems and to deduce the use of the potential function. The basic characteristics of those systems are uni-directional and multi-directional variables and the relevance of random processes. A very general mathematical approach that can represent these characteristics is the Fokker–Planck equation. We do not claim that this is the only possible way of modeling, we only claim that this is adequate for the cases of interest here. In the most general version we assume that 1) the state of the system is given at any time t by $\vec{x}(t)$ and $\vec{z}(t)$, 2) the value of $\vec{z}(t)$ (uni-directional variables), on average, increases in time, and 3) the dynamics of $\vec{x}(t)$ and $\vec{z}(t)$ contain stochastic elements. Furthermore, the system might be influenced by exogenous circumstances $\vec{\mu}(t)$, which might change in time. Such a system can be represented by the Fokker–Planck equation (Risken 1996):

$$\frac{\partial}{\partial t}P(\vec{y},t) = -\sum_{i=1}^{D}\frac{\partial}{\partial y_i}\left[A_i(\vec{y},\vec{\mu})P(\vec{y},t)\right] - \frac{1}{2}\sum_{i=1}^{D}\sum_{j=1}^{D}\frac{\partial^2}{\partial y_i \partial y_j}\left[B_{ij}(\vec{y},\vec{\mu})P(\vec{y},t)\right], \quad (1)$$

where $\vec{y}$ is the N-dimensional vector that describes the state of the system consisting of $\vec{x}$, and $\vec{z}$, and $P(\vec{y},t)$ denotes the probability to find the system in state $\vec{y}$ at time t. $A_i(\vec{y},\vec{\mu})$ and $B_{ij}(\vec{y},\vec{\mu})$ are functions describing the dynamics of the system dependent on its state and the exogenous circumstances. Besides the assumptions above, the use of the Fokker–Planck equation introduces one additional assumption on the dynamics of the system. It assumes that the random processes lead to normally distributed changes in the variables. We accept this additional assumption because the exact distribution of random changes is not relevant for the classification of path-dependent processes.

Although the literature provides many options to solve the Fokker–Planck equation (e.g., Risken 1996; Jordan et al. 1998; Denisov et al. 2009) this is not the aim here. On the one hand, Eq. (1) cannot be solved without specifying the functions $A_i(\vec{y},\vec{\mu})$ and $B_{ij}(\vec{y},\vec{\mu})$, whereas we want to make statements about the possible dynamics of systems in general. On the other hand, solutions to Eq. (1) usually refer to stable probability distributions; however, we are interested in systems that change permanently (uni-directional dimension).

Hence, the Fokker–Planck equation is not introduced here to solve mathematical calculations, except for some illustrative examples. The Fokker–Planck equation is mainly consulted here as a mathematical basis for the use of potential functions to describe the dynamics of systems. Potential functions can be drawn for systems with maximum two variables, so that we restrict the generality of the model by assuming that there is only one multi-directional and one uni-directional variable. This restriction has little impact on the potential developments of the systems, except on path-furrowing, which is discussed in more detail in the respective subsection. Assuming that $\vec{x}$ and $\vec{z}$ have only one dimension, Eq. (1) can be written as:

$$\begin{aligned}\frac{\partial}{\partial t}P(\vec{x},\vec{z},t) = &-\sum_{i=1}^{N_x}\frac{\partial}{\partial x_i}\left[A_{x,i}(\vec{x},\vec{z},\vec{\mu})P(\vec{x},\vec{z},t)\right] - \sum_{i=1}^{N_z}\frac{\partial}{\partial z_i}\left[A_{z,i}(\vec{x},\vec{z},\vec{\mu})P(\vec{x},\vec{z},t)\right] \\ &- \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2}{\partial y_i \partial y_j}\left[B_{ij}(\vec{y},\vec{\mu})P(\vec{y},t)\right].\end{aligned} \quad (2)$$

One-dimensional Fokker–Planck equations (assuming y to be the only variable) can be solved by defining a so-called potential V satisfying

$$\frac{-\partial}{\partial y}V(y) = A_y(y).$$

In two- and multi-dimensional cases, such a potential function V exists only under restrictive conditions that generally are not given. In our case, we always have at least two dimensions: x and z. Furthermore, given that in our case there is, on average, one variable that increases permanently with time, a stationary solution is not of interest. We instead focus on the analysis of the dynamics. This implies that a mathematical solution is neither feasible nor helpful for the intention of this paper.

However, if we keep variable z constant for a fictive moment, we can describe the dynamics in the x-dimension by a one-dimensional Fokker–Planck equation and the respective potential function (Risken 1996; Denisov et al. 2009). We use this assumption and the graphs of the potential function in our approach. Therefore, we analyze a system that is described by one variable x and the following Fokker–Planck equation in more detail:

$$\frac{\partial}{\partial t}P(x,t) = \frac{-\partial}{\partial x}[A(x)P(x,t)] - \frac{1}{2}\frac{\partial^2}{\partial x \partial x}[B(x)P(x,t)]. \tag{3}$$

For this equation the potential function V is given by

$$\frac{-\partial}{\partial x}V(x) = \frac{A_x(x)}{B(x)}, \tag{4}$$

and the long-term stationary solution of the Fokker–Planck equation is given by (Denisov et al. 2009)

$$P_{\mathrm{st}}(x) = \frac{-N}{B(x)}\exp\left(2V(x)\right). \tag{5}$$

To demonstrate the potential function, we can use the following example and assume that the system is driven by a force that tries to bring it always back to $x = 0$. Technical examples for this are a spring or a pendulum. In such a case, the drift term in the Fokker–Planck equation is given by $A(x) = -ax$. Assuming the strength of randomness $B(x)$ being constant (for simplicity = 1), the potential function is

$$V(x) = \frac{-1}{2}ax^2. \tag{6}$$

This potential function is depicted in Fig. 1a (for $a = 0.5$), while the stationary solution for the example using Eq. (6) is depicted in Fig. 1b (for $B(x) = 1$). Since the derivative of the potential function represents the average dynamics of the system (see Eq. 4), we can deduce the dynamics of the system from the slope of the potential function. On average, the system will develop in the downhill direction. The slope of the potential function represents the speed of the development $A(x)$ divided by the extent of randomness $B(x)$. Consequently, a steeper slope does not mean that the system develops faster in this direction but that the likelihood of a development in the opposite direction is smaller because of randomness. Hence, higher hills imply a smaller likelihood for the system to climb them.

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