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Changes in the gender pay gap over time: the case of West Germany

Marina Bonaccolto-Töpfer^{1*}, Carolina Castagnetti² and Luisa Rosti²

Abstract

Using data from the German Socio-Economic Panel, this paper analyzes changes in the gender pay gap in West Germany between 1984 and 2020. The literature generally observes a catching-up of women over time with a slowdown since the mid-1990s and often concentrates on the USA. We present both an aggregate and detailed decomposition of changes in wages allowing us to directly test for changes in the components of the decomposition across gender and time. Apart from standard OLS, we use linear unconditional quantile regressions in order to be able to take changes in the gap and its components at the mean and across the distribution into account. We find that the gender pay gap statistically significantly declined at the bottom and the middle, while it increased at the top of the wage distribution. These results suggest that glass ceiling is a major challenge to the West German labour market.

Keywords Changes in the gender pay gap, Decomposition, Unconditional quantile regression

JEL Classification J7, J13, J310

1 Introduction

There are many relevant economic questions that may be answered by decomposing changes in pay gaps in its components (explained and unexplained). Gender Pay Gaps (GPGs), for instance, while declining in many countries in the 1970s and 1980s, have shown a slowdown in convergence in the 1990s (e.g. Goldin 2014; Blau et al. 2017; Bruns 2019). Blau et al. (2017) attribute the 1990s slowdown in the GPG convergence to a relative persistent unexplained part. This part includes, inter alia, labour market selectivity, unmeasured characteristics and labour market discrimination. The related literature finds typically substantial differences in changes across the distribution (e.g. Antonczyk et al. 2010; Arellano and Bonhomme 2017; Gallego Granados and Wrohlich 2019;

Maasoumi and Wang 2019). Applied research has also observed that women are 'swimming upstream' (Blau and Kahn 1997). The latter refers to the persistence of the GPG despite diminishing gaps in characteristics between men and women over time. These findings suggest that differences in skill prices between men and women play a major role for the slowing convergence of the GPG. The example shows that identifying the determinants (such as gender differences in skill prices) of changes in GPGs over time is pivotal for efficient policy measures and a major contribution of this paper.

In this paper, we adapt the Oaxaca-Blinder (OB) decomposition (Blinder 1973; Oaxaca 1973) for estimating and decomposing changes in the West German GPG between two time periods or points in time. Similar to Gelbach (2016), we use the omitted variable bias formula for the adapted decomposition. Consequently, the approach relies on a single OLS regression and standard errors for the different components of the decomposition are easily obtained. That is, the approach allows us to draw inference on changes in pay gaps as well as on the different components between groups and across time.

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An advantage of our decomposition is its simplicity in terms of implementation. As our method follows the intuition of OB—by decomposing the change in the GPG in an explained and unexplained part—it relies on the intuitive and probably most used decomposition in the literature. In addition, it allows for both aggregate and detailed decompositions. Further, we use linear unconditional quantile regressions or Recentered Influence Function (RIF) OLS (Firpo et al. 2009) in order to extend the decomposition beyond the mean. Using unconditional quantile regression decompositions is computationally easy, especially compared to conditional quantile decompositions, and permits us to calculate detailed OB-type decompositions of both the explained and unexplained part along the distribution. Moreover, changes in inequality, measured by inter-quantile gaps, can be easily derived.2

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Using data from the German Socio-Economic Panel (SOEP), we analyse the GPG at two different and distant points in time: 1984 and 2020. The results show a substantial and statistically significant reduction in the GPG over the last 36 years for most parts of the wage distribution. At the top, however, the gap increased statistically significantly. This result implies that the gap evolves differently across the wage distribution. The catching-up of women in terms of educational attainment contributed only slightly to the convergence of the gap over time. A main driver of the change are differences in prices for labour market characteristics such as full-time experience and job tenure between men and women.

Gallego Granados and Wrohlich (2019) provide a distributional analysis of the GPG in West Germany over the period 1990–2014 using the SOEP. The paper takes (changing) sample selection in full-time employment via wage imputation into account. Gallego Granados and Wrohlich (2019) find slight convergence of the GPG during the 1990s and stagnation thereafter for all points of the wage distribution. Biewen et al. (2018) using administrative data look at the rise of wage inequality in West Germany (1985-2000) taking changes in non- and parttime employment among full-time employees over time into account. Their paper looks separately at men and women and finds substantial differences across gender and over time. Bruns (2019) looks at the evolution of the West German GPG since the 1990s. Based on administrative linked-employer-employee data, the author identifies firm characteristics as main driver of the slow-down in convergence of the gap since the mid 1990s. The finding of slowing convergence in the West German GPG is in line with results of e.g. Blau et al. (2017) for the USA. In particular, the results from the literature suggest that gender wage inequality is far from being solved and that the gap differs substantially across the distribution.

We contribute to this literature in the following ways. First, we propose an easy-to-use and easy-to-implement method that is based on the popular OB interpretation. Second, the approach allows to decompose the change in the GPG on aggregate and in detail. Third, the decomposition can be applied along the wage distribution. A further contribution of our paper is that it adds to the literature on changes in the GPG in West Germany.

The remainder of this paper proceeds as follows. Section 2 provides an overview of two prominent traditional methods used to estimate changes in wage gaps over time. Section 3 outlines the proposed adaptation of the OB decomposition, while Sect. 4 discusses inference issues and the derivation of the detailed decomposition. Section 5 presents the empirical application and Sect. 6 concludes.

2 Changes in wage gaps: traditional methods in use in the literature

The OB approach can be considered the 'workhorse' in empirical labour economics when it comes to decomposition methods (Fortin et al. 2011). The idea of the OB decomposition is to construct a counterfactual such that the gap can be attributed to a characteristics (explained) and a prices (unexplained) component. In case of GPGs, the OB decomposition estimates Mincer-type wage equations separately for men and women and then decomposes the wage differential into the two components.

Smith and Welch (1989) propose the so-called double OB decomposition for the case of changes in wage gaps over time. The decomposition reads as:

$$\begin{split} \Delta &= [(\bar{x}_{ME}^{'} - \bar{x}_{FE}^{'}) - (\bar{x}_{MS}^{'} - \bar{x}_{FS}^{'})]\hat{\beta}_{MS} \\ &+ (\bar{x}_{ME}^{'} - \bar{x}_{FE}^{'})(\hat{\beta}_{ME} - \hat{\beta}_{MS}) \\ &+ \bar{x}_{FE}^{'}[(\hat{\beta}_{ME} - \hat{\beta}_{FE}) - (\hat{\beta}_{MS} - \hat{\beta}_{FS})] \\ &+ (\bar{x}_{FE}^{'} - \bar{x}_{FS}^{'})(\hat{\beta}_{MS} - \hat{\beta}_{FS}) \end{split} \tag{1}$$

where Δ refers to the change in the wage gap over time and is defined as follows: $\Delta = (\bar{y}_{ME} - \bar{y}_{FE}) - (\bar{y}_{MS} - \bar{y}_{FS})$ where \bar{y}_{GT} is the dependent variable (log of hourly wages in our case) of group G = M, F (with M = male and F = female) at time T = S, E (with E and S referring to the ending (2020) and starting period (1984), respectively).

The first line of the decomposition contains the explained component which is divided in a main quantity

 $^{^1}$ Blinder (1973) and Oaxaca (1973) were together cited more than 20,000 times as of 10 May, 2022, according to the Google Scholar citation statistics.

² Inter-quantile gaps allow for example the analysis of 'glass ceiling' and 'sticky floors' in gender economics. That is, particularly pronounced gaps at the top and bottom of the distribution, respectively (Arulampalam et al. 2007, e.g.).

effect that shows how the wage gap changed because men and women became more similar or dissimilar in characteristics, and a secondary quantity effect attributable to changes in the reference wage structure over time. The second line represents the price effect that is split in a primary price effect, i.e. the effect of changes in the wage structure between men and women over time, and a secondary price effect attributable to changes in reference endowments over time.

Juhn et al. (1991) (JMP) proposed a decomposition for changes in residual wage differentials that considers the effect of unobserved skills on the gap. However, unlike OB-type decompositions, the JMP method yields unexplained and explained components only at an aggregate level and, hence, does not provide a detailed decomposition of the variation in the gap between groups over time. Further, as Suen (1997) stressed, the JMP decomposition of wage residuals into standard deviations (the price of unobserved skills) and percentile ranks (the level of unobserved skills) is unbiased only when percentile ranks are independent of the standard deviation.

More recently, a growing body of research focuses on changes of the GPG over time (Blau and Kahn 2006; Olivetti and Petrongolo 2008; Mulligan and Rubinstein 2008; Goldin 2014; Arellano and Bonhomme 2017; Maasoumi and Wang 2019). This literature generally observes a catching-up of women over time with a slow-down since the mid-1990s and mainly concentrates on the USA (among others Blau and Kahn 2000; Blau et al. 2017; Mulligan and Rubinstein 2008; Goldin 2014; Maasoumi and Wang 2019) or in fewer cases on the UK (Blundell et al. 2007; Arellano and Bonhomme 2017). This paper adds to the literature on changes in the GPG over time in West Germany (Biewen et al. 2018; Gallego Granados and Wrohlich 2019; Bruns 2019; Schmitt and Auspurg 2022) by providing an approach that is easy-to-use and -interpret. Moreover, we present the detailed and aggregate decomposition at the mean as well as across the wage distribution.

3 Our estimation method

In this paper we use a double OB decomposition. Differently from Smith and Welch (1989), who distinguish between main and secondary price and quantity effects, we retain the original OB interpretation of explained and unexplained components. Moreover, our approach delivers both aggregate and detailed decomposition components. The detailed decomposition allows us to identify potential drivers of changes in pay gaps over time such as educational attainment, labour market presence or occupational and sector sorting.

To estimate the joint model, we distinguish (as in Gelbach (2016)) between two sets of regressors, X_1 and X_2 . X_1 represents the regressors of the base specification containing only a constant, an interaction term between the gender and time dummy, and the gender as well as the time dummies themselves:

$$X_1 = \begin{bmatrix} 1, FS, F, S \end{bmatrix}$$

where *F* is a dummy variable equal to 1 for a female and 0 for a male, and S is a dummy variable equal to 1 for the starting period and equal to zero for the ending period. The second set of regressors, X_2 , of dimension $(N \times 4K)$, contains the matrix of characteristics X and the interactions of the gender and time dummy with *X*:

$$X_2 = [X, FX, SX, FSX] \tag{2}$$

where FX and SX are the interaction variables between the regressors X, the female dummy F and the starting period dummy S, respectively. FSX represents the interaction variable among regressors X and dummy variables F and S.

The base model is defined as the model that only considers the set of regressors X_1 :

$$y = \alpha_0^{base} + FS\alpha_1^{base} + F\alpha_2^{base} + S\alpha_3^{base} + \epsilon^{base}$$
 (3)

The full model is defined as the model that considers both set of regressors, X_1 and X_2 :

$$y = \alpha_0^{full} + FS\alpha_1^{full} + F\alpha_2^{full} + S\alpha_3^{full} + X\beta_1$$

+ $FX\beta_2 + SX\beta_3 + FSX\beta_4 + \epsilon^{full}$ (4)

By considering the set of regressors X_2 as omitted variables, the OVB formula implies:

$$\hat{\alpha}^{base} = \hat{\alpha}^{full} + (X_1'X_1)^{-1}X_1'X_2\hat{\beta}^{full}$$
 (5)

where $(X_1'X_1)^{-1}X_1'X_2$ is the linear projection of X_2 on X_1 and $\hat{\beta}^{full}$ is the vector of estimated coefficients β from the full model (4). Model (5) can be decomposed as follows:

$$\hat{\alpha}^{base} = \hat{\alpha}^{full} + \hat{\delta} \tag{6}$$

where $\hat{\delta} = (X_1'X_1)^{-1}X_1'X_2\hat{\beta}^{full}$ and

$$\hat{\delta} = \hat{\delta}_X + \hat{\delta}_{FX} + \hat{\delta}_{SX} + \hat{\delta}_{FSX} \tag{7}$$

with $\hat{\delta}_J = \hat{\Gamma}^J \hat{\beta}_J^{full}$, with $\hat{\Gamma}^J = (X_1'X_1)^{-1}X_1'J$ and J is the portion of the matrix (2) that corresponds to the set of regressors *J*, for $J = X, \dots, FSX$ in (2).³

We are interested in the estimation and decomposition of the GPG across two periods, E and S:

³ Accordingly, $\hat{\delta}_X = \hat{\Gamma}^X \hat{\beta}_X^{\text{full}}$, with $\hat{\Gamma}^X = (X_1'X_1)^{-1}X_1'X$ of dimension $(4 \times K)$ and $\hat{\beta}_{X}^{full}$ is the $(K \times 1)$ vector $\hat{\beta}_{1}$ in Eq. (4).

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$$\Delta = \Delta_E - \Delta_S = \left(\bar{y}_{ME} - \bar{y}_{FE}\right) - \left(\bar{y}_{MS} - \bar{y}_{FS}\right)$$

with (Δ_E, Δ_S) being the gap in the outcome variable y in period E and S, respectively. It can be easily shown that:

$$\Delta_E = \left(ar{y}_{ME} - ar{y}_{FE}
ight) = -\hat{lpha}_2^{base}$$

and

$$\Delta_S = \left(ar{y}_{MS} - ar{y}_{FS}
ight) = -\hat{lpha}_1^{\it base} - \hat{lpha}_2^{\it base}$$

Then $\hat{\alpha}_1$ is the change in the gender wage differential over time:

$$\Delta_E - \Delta_S = \hat{\alpha}_1^{base}$$

Hence, given Eq. (3), we decompose $\hat{\alpha}_1^{\textit{base}}$ according to (5) or (6).

The second row of Eq. (5) (i.e. the change in the wage gap evaluated at the mean) can be re-written as a double OB decomposition in the following way:

$$\alpha_1^{base} - \alpha_1^{full} = \underbrace{(\hat{Q}_E + K)}_{\hat{\delta}_1} + \underbrace{(\hat{P}_E + W)}_{\hat{\delta}_2} + \underbrace{(-\hat{Q}_S - K)}_{\hat{\delta}_3} + \underbrace{(-\hat{P}_S - W)}_{\hat{\delta}_4}$$
(8)

where $\hat{Q}_S = (\bar{x}'_{MS} - \bar{x}'_{FS})\hat{\beta}_{MS}$, is the estimated explained component and $\hat{P}_S = \bar{x}'_{FS}(\hat{\beta}_{MS} - \hat{\beta}_{FS})$, the estimated unexplained component in period S, and $\hat{Q}_E = (\bar{x}'_{ME} - \bar{x}'_{FE})\hat{\beta}_{ME}$, and $\hat{P}_E = \bar{x}'_{FE}(\hat{\beta}_{ME} - \hat{\beta}_{FE})$, the estimated explained and unexplained component in period E, respectively. We are interested in $(\hat{Q}_E - \hat{Q}_S)$ and $(\hat{P}_E - \hat{P}_S)$ corresponding to the change in the explained and unexplained component over time, respectively. Note that in Eq. (8) the unexplained part does not include intercept terms. The terms K and W cancel out. The term K $((\bar{x}'_{FS} - \bar{x}'_{MS})\hat{\beta}_{ME})$ is the difference in observed characteristics by gender in the base year evaluated at end-year male parameter values. The second term W, $(\bar{x}'_{FS}(\hat{\beta}_{FE} - \hat{\beta}_{ME}))$, represents gender differences in estimated coefficients in the end-year weighted for female characteristics in the starting year.

 $\hat{\alpha_1}^{full}$ represents the component of the change in the GPG over time (Δ) that cannot be explained by the quantity and the price effect. That is, it represents differences in intercepts by gender and over time. In the OB

decomposition, the intercept terms play an important role. Group differences in intercepts are generally attributed to the unexplained part and are often referred to as the group difference in starting points. Blinder (1973) called this part the unexplained part of discrimination, as interpretation of the difference in the intercepts may not be straightforward.⁵ Instead of attributing the difference in the intercepts to the price component, we focus the analysis on the components that can be attributed to either of the two parts of the decomposition. That is, we focus on differences in characteristics (explained component) and prices (unexplained component).

The above decomposition approach can be easily extended beyond the mean by using the linear unconditional quantile regression model (RIF-OLS) introduced by Firpo et al. (2009). In case of estimation beyond the mean, instead of using y as dependent variable, we use the Recentered Influence Function (RIF) of γ at the unconditional quantile Q_{τ} ; RIF $(\gamma; Q_{\tau})$ (see Appendix A for additional details on the RIF-OLS approach). The RIF-OLS approach allows for the unconditional mean interpretation that we need in OB-type decompositions (Fortin et al. 2011). This property represents a major advantage over the conditional quantile approach that does not allow for the unconditional mean interpretation. Further, decompositions based on RIFs allow us to conduct detailed decompositions of both the explained and unexplained component. They are also computationally easy compared to the often-used Machado and Mata (2005) decomposition in case of conditional quantiles.

To sum up, the double OB decomposition can be obtained as follows:

- 1. Estimate the base (Eq. (3)) and full (Eq. (4)) model using least squares.
- 2. The estimated coefficient $\hat{\alpha}_1^{base}$ of the interaction term between gender and time (*FS*) in the base model gives Δ , i.e. the change in the GPG over time.
- 3. The part of the change in the GPG over time that can be attributed to changes in the explained and unexplained component is given by $\hat{\alpha}_1^{base} \hat{\alpha}_1^{full}$, where $\hat{\alpha}_1^{full}$ is the estimated coefficient of the interaction term between gender and time (FS) from the full model.
- 4. $\Delta = \hat{\alpha}_1^{base} \hat{\alpha}_1^{full}$ can be decomposed in changes in the explained and unexplained part $(\hat{\delta}_1 + \hat{\delta}_3)$ and $\hat{\delta}_2 + \hat{\delta}_4$, respectively) using the OVB formula and treating the controls of the full model as omitted.

⁴ Observe that $(\hat{Q}_E - \hat{Q}_S)$ and $(\hat{P}_E - \hat{P}_S)$ is equivalent to $(\hat{\delta}_1 + \hat{\delta}_3)$ and $(\hat{\delta}_2 + \hat{\delta}_4)$, respectively.

 $^{^{5}}$ According to Jones (1983), the problem is so critical that the intercept term is not interpretable.

5. Each term $(\hat{\delta}_1 + \hat{\delta}_3)$ and $(\hat{\delta}_2 + \hat{\delta}_4)$ can be decomposed into its single components to obtain a detailed decomposition.

For the extension to unconditional quantiles, use the RIF of the log wage at Q_{τ} as dependent variable and follow the steps listed above.

4 Inference and detailed decomposition

The asymptotic distribution of $\sqrt{N}(\hat{\delta}-\delta)$, with $\hat{\delta}=(\hat{\delta}_1,\dots,\hat{\delta}_4)$ is presented in Appendix B. Given the distribution of the parameters $\hat{\delta}$, the proposed decomposition allows us to draw inference about the dynamic of the single components of the GPG. For instance, we may want to investigate whether the convergence of the change in the wage gap, Δ , can be explained by the convergence of the level of endowments (explained component) or by changes in prices (unexplained component). The hypothesis that the convergence is driven by changes in observed characteristics is tested by:

$$H_0: \delta_1 + \delta_3 = 0$$

which is equivalent to testing the hypothesis that there was no change in endowments between group M and group F over time:

$$H_0: Q_S = Q_E$$

Analogously, changes in the prices between M and F are analysed by testing whether the components of the price effects have been stable over time:

$$H_0: \delta_2 + \delta_4 = 0$$

which is equivalent to testing:

$$H_0: P_S = P_E$$

where the null hypothesis indicates no change in prices between M and F over time. Moreover, each $\hat{\delta}$ can be decomposed into its single components. Researchers and politicians may be interested in investigating whether the convergence of the GPG was driven by skill-related human capital attributes in terms of rising prices or of rising endowment levels. For instance, the contribution of labour market experience to the quantity component in period S can be extracted from $\hat{\delta}_3$. Because each $\hat{\delta}$ is given by

$$\hat{\delta}_i = \sum_{k=1}^K \hat{\delta}_{ik} \text{ for } i = 1, \dots, 4$$
(9)

where K are the regressors considered in the analysis. Observe that inference can be easily extended to the single components, i.e. to the detailed decomposition. The

detailed decomposition allows to consistently identify the drivers of the wage gap (conditional on having included all relevant controls).

5 Empirical application

In this section, we present the data set used and show results of the proposed double OB decomposition. After the data description (Sect. 5.1), we confront insights from results of the (aggregate) standard OB decomposition with the corresponding outcomes of the double approach suggested in this paper (Sect. 5.2). Finally, we look at the detailed decomposition of our double decomposition (Sect. 5.3). We use men (for the standard OB) or men in 1984 (for our double decomposition) as reference category. As the results may change depending on the choice of the non-discriminatory wage structure, we show the estimation outcome of the aggregate decomposition using women in 1984 as reference category in Appendix C.

5.1 Data

We illustrate our decomposition by an empirical application on the variation of the GPG between 1984 and 2020 in West Germany using data from the 1984 and 2020 waves of the German Socio-Economic Panel (SOEP).⁶ The SOEP is a representative annual household panel survey on households and individuals living in Germany.

We restrict the analysis to West Germany as the aggregate GPG differs considerably between East and West Germany: it is substantially lower in East compared to West Germany. For instance, in 2020 the GPG amounted to 6% in East compared to 20% in West Germany (Destatis 2022). Moreover, the composition of the gap differs substantially between the Eastern and the Western part of Germany (e.g. Hirsch 2013; Boll et al. 2014; Hunt 2002). Reasons for these deviations lie inter alia in lower female labour force participation and a stronger focus on traditional gender roles in West Germany (Boll et al. 2014). As it is difficult to account for these differences, we restrict the analysis to West Germany. Further, concentrating on West Germany allows for a better comparison with the related literature (Gallego Granados and Wrohlich 2019; Biewen et al. 2018; Antonczyk et al. 2010; Bruns 2019).

The dependent variable is the natural logarithm of hourly gross earnings (in 2015-prices). We calculate this variable based on monthly gross earnings and actual weekly working hours. We calculate the RIFs based on log hourly wages adjusted for the survey years in order

⁶ SOEP (2022), version 37, see https://www.diw.de/sixcms/detail.php?id=diw_01.c.838578.de (accessed 2022-10-14) for details. See Goebel et al. (2019) for a description of the data set.

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to account for potential changes in the wage distribution over time (DiNardo et al. 1996; Bonaccolto-Töpfer and Briel 2022). Following the standard GPGs literature, we use quadratic polynomials of actual full-time experience as well as variables indicating job tenure and past parttime experience as explanatory variables. We define parttime employment as working less than 30 h per week. We add controls for the highest educational degree as well as age categories. Moreover, we control for migration background and marital status and include dummies for working in a public-sector firm as well as firm size. In order to account for the motherhood penalty (Dougherty 2006; Gangl and Ziefle 2009), we include controls for having at least one child and having small children (children smaller than three years and between three and six years, respectively, similar to e.g. Dougherty (2006) and Arellano and Bonhomme (2017)). Further, we include federal state dummies as well as occupation and industry or sector dummies. We use the ISCO88 (1-digit) for classification of occupations and NACE (level 1) for industries or sectors. Our analysis focuses on full-time employees between 16 and 67 years of age. Dropping missing values in relevant control variables leaves us with a final sample size of 8,202 men and women (4,064 in 1984 and 4,138 in 2020). We exclude armed forces (ISCO) and activities of extraterritorial organisations and bodies (NACE) due to few female observations per group (< 5). Similarly, we drop skilled agricultural, forestry and fishery workers.

Table 1 reports mean and standard deviation of selected variables. Panel (a) represents the corresponding figures for the starting period 1984 and panel (b) for the ending period 2020. Men earned on average 32.7% (log approximation) more than women in 1984. In 2020, the gap shrank to 16.6% (log approximation). That is, the aggregate wage difference was on average 16 percentage points higher in 1984 than in 2020. While women are younger than their male colleagues in both years, this difference is declining over time. Women have higher educational attainment (Abitur) than men in 2020 but not in 1984. Men outperform their female colleagues in terms of labour market characteristics like full-time experience and job market tenure in both years.

Women have more past experience in part-time work than men in both years. Yet, while women have about one year more part-time experience compared to men in 1984, this difference has increased over time by four years. The latter may be due to relatively more part-time employment in 2020 compared to 1984 being pre-dominately done by women. This descriptive finding is in line with results for Germany of increasing incidence of part-time work over time in general and in particular for women (Paul 2016; Tamm et al. 2017; Biewen et al. 2018). Further, it is in line with official statistics

on part-time employment (as % of total employment) of the OECD⁷: in 2020 37% (10%) of women (men) were part-time employed, while in 1984 only 24% (5.6%) of women (men) were part-time employed. That is, the proportion of part-timers, though increasing for both men and women, rose especially for females. The importance of part-time work for the GPG is also emphasized by Schmitt and Auspurg (2022) who analyse the impact on the increasing supply of non-standard working hours on the GPG in West Germany for the period 1985–2014 using the SOEP. They find that the large increase in part-time work among women in combination with increasing wage gaps between part- and full-time workers substantially widened the GPG and offset the equalizing effects of declining gender gaps in human capital.

Significantly more men than women are married in both years. We observe more females than males in small and medium firms in both years, while the opposite holds for large firms. The finding that women are less often employed in larger firms and that this relation is rather stable over time is in line with findings of Bruns (2019). Finally, we observe a substantial increase in the femalemale ratio in our sample from 42.4% in 1984 to 57.8% in 2020. The latter is in accordance with an increasing labour force participation of women over time in Germany.

5.2 Aggregate decomposition

Our empirical application analyzes the change of the GPG from 1984 to 2020 in West Germany. In addition to looking at the average change in the pay gaps over time, we extend our proposed decomposition along the wage distribution using linear unconditional quantile regression (RIF-OLS). To conduct the proposed double decomposition, we use Gelbach (2014)'s Stata code *b1x2*. The estimation results are presented as follows. First, we look at the traditional estimation outcome, i.e. OB decompositions of the wage gap in 1984 and 2020 separately. For simplicity, we focus here on the aggregate decomposition. Second, we look at the results from the suggested double decomposition both aggregate and detailed.⁸

Table 2 shows the traditional OB decomposition with men as reference category. In line with the literature, we find differences in the gap over time and at various points of the distribution (e.g. Arulampalam et al. 2007; Albrecht et al. 2009; Gallego Granados and Wrohlich 2019). This result suggests that the change in the wage gap over time is not evenly distributed across the wage

⁷ https://data.oecd.org/emp/part-time-employment-rate.htm, accessed 2022-10-24.

 $^{^{8}}$ In Table 4 in the Appendix, we represent the base and full regressions at the mean.

 Table 1
 Descriptive statistics for men and women in 1984 and 2020, selected controls

Variable	(1)			(3) (4)	
	Men		Women		Difference
	Mean	Std. dev.	Mean	Std. dev.	
Panel (a): 1984					
Log gross hourly wage	1.707	0.426	1.380	0.530	0.327
Young 16–29 years	0.214	0.410	0.414	0.493	- 0.200
Adult 30–39 years	0.277	0.448	0.231	0.422	0.046
Adult 40–49 years	0.307	0.461	0.238	0.426	0.069
Old 50–65 years	0.231	0.421	0.140	0.347	0.091
Full-time experience (in years)	19.409	11.489	12.268	10.175	7.141
Past part-time experience (in years)	0.152	0.985	1.402	4.040	- 1.250
Tenure (in years)	12.496	9.846	8.131	7.498	4.365
Basic secondary education (Hauptschule)	0.613	0.487	0.495	0.500	0.118
Secondary education (Realschule)	0.158	0.365	0.298	0.458	- 0.140
Upper secondary education (Abitur)	0.147	0.354	0.130	0.336	0.017
Other degree	0.054	0.226	0.040	0.196	0.014
No degree	0.029	0.167	0.037	0.188	- 0.008
Married	0.736	0.441	0.476	0.500	0.260
Migration	0.126	0.332	0.125	0.331	0.001
Public sector	0.265	0.441	0.308	0.462	- 0.043
Small firm <200 employees	0.146	0.353	0.238	0.426	- 0.092
Medium firm 200–1999 employees	0.279	0.448	0.284	0.451	- 0.005
Large firm > 1999 employees	0.575	0.494	0.477	0.500	0.098
At least one child	0.685	0.465	0.411	0.492	0.274
Children < 3 years	0.114	0.318	0.067	0.250	0.047
Children between 3 and 6 years	0.097	0.296	0.031	0.173	0.066
Observations	2855		1209		4064
Panel (b): 2020					
Log gross hourly wage	3.220	0.427	3.054	0.402	0.166
Young 16–29 years	0.105	0.307	0.121	0.326	- 0.016
Adult 30–39 years	0.286	0.452	0.261	0.439	0.025
Adult 40–49 years	0.232	0.422	0.244	0.430	- 0.012
Old 50–65 years	0.403	0.491	0.398	0.490	0.005
Full-time experience (in years)	20.215	12.272	14.226	10.872	5.989
Past part-time experience (in years)	1.184	2.703	5.472	6.972	- 4.288
Tenure (in years)	12.933	11.676	11.604	10.713	1.329
Basic secondary education (Hauptschule)	0.218	0.413	0.136	0.343	0.082
Secondary education (<i>Realschule</i>)	0.249	0.433	0.278	0.448	- 0.029
Upper secondary education (Abitur)	0.379	0.485	0.435	0.496	- 0.056
Other degree	0.112	0.316	0.088	0.283	0.024
No degree	0.041	0.199	0.063	0.244	- 0.022
Married	0.572	0.495	0.397	0.489	0.175
Migration	0.248	0.432	0.229	0.420	0.019
Public sector	0.176	0.381	0.331	0.471	- 0.155
Small firm < 200 employees	0.170	0.321	0.149	0.356	- 0.032
Medium firm 200–1999 employees	0.221	0.415	0.231	0.422	- 0.010
Large firm > 1999 employees	0.662	0.473	0.620	0.486	0.042
At least one child	0.326	0.469	0.260	0.439	0.066

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Table 1 (continued)

Variable	(1)	(2)	(3)	(4)	(5)
	Men		Women		Difference
	Mean	Std. dev.	Mean	Std. dev.	
Children < 3 years	0.135	0.342	0.064	0.245	0.071
Children between 3 and 6 years	0.071	0.258	0.041	0.199	0.030
Observations	2623		1515		4138

Reported differences are based on a regression of the selected variable on a male dummy.* * *p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors are used. Dummy variables if not indicated differently. Gross hourly wages are in 2015-prices. SOEP sample weights used. Source: SOEP v37

distribution. The GPG declined at the mean (as mentioned before, cfr. Table 1) as well as at the bottom and in the middle of the wage distribution, while the gap at the top of the distribution increased between 1984 and 2020. In all cases both components are statistically significant in 1984 as well as in 2020 (except the unexplained part in 2020 at the 10th percentile). The component due to gender differences in observable characteristics decreased (explained part) at the mean and median but increased at the bottom and top. Therefore, one may conclude that bottom- and top-income women did not succeed in catching-up to their male counterparts in terms of general observable characteristics, while women at the mean and median did successfully catch up. Differences in prices between men and women decreased (unexplained part) at all points – except the top. This result suggests that equal-pay measures contributed to a closing of the gap since the 1980s in West Germany only at the bottom, mean and median. However, as typically—and as in our Table 2—in regression tables standard errors (and not e.g. confidence bands) are reported. Without further calculations, the drawn conclusions are therefore not based on statistical tests.

Table 3 shows the change of the GPG in West Germany between 1984 and 2020 at the mean and across the wage distribution. Recall that the components in the double decomposition consist in testing for $Q_E = Q_S$ and $P_E = P_S$. That is, we test whether the quantity or price component, respectively, has changed over time (from the starting period S to the ending period E) as outlined in Sect. 4. We find substantial differences in changes over time of the GPG along the wage distribution. While the gap increased at the top, it decreased substantially at the mean, bottom and median of the wage distribution. The reduction is most pronounced at the bottom. This finding implies that the gap was higher in 2020 than in 1984 only

Table 2 Oaxaca–Blinder aggregate decomposition in 1984 and 2020 at the mean and selected percentiles

	Mean		10th Percer	h Percentile 50th Percent		tile	90th Percentile	
	(1) (2)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	1984	2020	1984	2020	1984	2020	1984	2020
GPG	0.327*** (0.020)	0.166*** (0.018)	0.533*** (0.037)	0.191*** (0.040)	0.305*** (0.016)	0.145*** (0.020)	0.175*** (0.024)	0.239*** (0.030)
Explained: Q	0.166*** (0.031)	0.093*** (0.020)	0.148*** (0.035)	0.162*** (0.044)	0.157*** (0.019)	0.049** (0.022)	0.104*** (0.031)	0.128*** (0.046)
in % of GPG	50.76	56.02	27.77	84.82	51.48	33.79	59.43	53.56
Unexplained: P	0.161*** (0.036)	0.073*** (0.024)	0.385*** (0.052)	0.029 (0.056)	0.148*** (0.025)	0.096*** (0.027)	0.071** (0.036)	0.110** (0.048)
in % of GPG	49.24	43.98	72.23	15.18	48.52	66.21	40.57	46.44
Observations	4064	4138	4064	4138	4064	4138	4064	4138
Unexplained: P	0.161*** (0.036)	0.073*** (0.024)	0.385*** (0.052)	0.029 (0.056)	0.148*** (0.025)	0.096*** (0.027)	0.071** (0.036)	0.110** (0.048)
in % of GPG	49.24	43.98	72.23	15.18	48.52	66.21	40.57	46.44
Observations	4064	4138	4064	4138	4064	4138	4064	4138

Table shows aggregate Oaxaca–Blinder decompositions of the Gender Pay Gap (GPG) in West Germany. Men are used as reference category. The regressions include quadratic polynomials of full-time experience, job tenure, past part-time experience (controls in years), as well as dummies for highest educational attainment, age, marital status, migration background, having at least one child, having at least one child below the age of two and between 2 and 6 years, respectively. Further, the following dummies are included: occupations (ISCO-88, 1-digit) and industries (NACE, level 1), small (< 200 employees) and medium (200 - 1999 employees) firm size and public-sector firm. Federal state fixed effects included. Robust (for the mean) and bootstrapped (for the estimates beyond the mean, 500 replications) standard errors in parentheses. ** ***p < 0.01, ***p < 0.01, ***p < 0.05, **p < 0.1. SOEP sample weights used. Source: SOEP v37

Table 3 Aggregate double decomposition of the change in the explained component and the gender pay gap (GPG) 2020-1984 at the mean and selected percentiles

	(1) 2020-1984	(2) 2020-1984	(3) 2020-1984	(4) 2020-1984
	Mean	RIF 10	RIF 50	RIF 90
Change in GPG: $\Delta = \hat{\alpha}_1^{base}$	- 0.161*** (0.029)	- 0.342*** (0.061)	- 0.159*** (0.029)	0.064* (0.038)
Change in Explained: $Q_E - Q_S$	- 0.073*** (0.024)	0.014 (0.058)	- 0.108*** (0.028)	0.024 (0.061)
in % of change in GPG	45.34	- 4.09	67.92	37.50
Change in Unexplained: $P_E - P_S$	- 0.267 (1.015)	0.732 (0.839)	- 0.592 (0.041)	- 0.058 (0.529)
in % of change in GPG	165.84	- 214.04	32.42	- 90.63
Change in Intercepts: $\hat{m{lpha}}_1^{full}$	0.179 (0.299)	- 1.088 (0.838)	0.540 (0.425)	0.098 (0.528)
in % of change in GPG	– 111.18	318.13	- 0.34	153.13
Observations	8202	8202	8202	8202

Figure shows aggregate double decomposition of changes in the Gender Pay Gap (GPG) in West Germany. Estimation based on approach outlined in Sect. 3. Men in 1984 used as reference category. Δ refers to the aggregate change in percentage points in the GPG 2020-1984 in West Germany. Change in Explained refers to the corresponding change in the quantity effect. Change in Unexplained refers to the change in the price effect. $\hat{\alpha}_1^{pase}$ and $\hat{\alpha}_1^{full}$ are the coefficient estimates of Female X Year from the base and full regression, respectively. Female X Year is an interaction between the female and 2020-year dummy. See Table 4 for the corresponding regressions at the mean. Set of controls includes: quadratic polynomials of full-time experience, job tenure, past part-time experience (controls in years), as well as dummies for highest educational attainment, age, marital status, migration background, having at least one child, having at least one child below the age of 2 years and between two and six years, respectively. Further, the following dummies are included: occupations (ISCO-88, 1-digit) and industries (NACE, level 1), small (< 200 employees) and medium (200 — 1999 employees) firm size as well as public-sector firm, respectively. Federal state fixed effects included. Robust (for the mean) and bootstrapped (for the estimates beyond the mean, 500 replications) standard errors in parentheses. ** ** p < 0.01, ** p < 0.05, ** p < 0.1. SOEP sample weights used. Source: SOEP v37

at the top. Yet, at other points the gap is lower in 2020 than in 1984. We immediately see that both the divergence at the top as well as the convergence at bottom, mean and median is statistically significant.

Table 3 shows that the part of the convergence explained by general observable characteristics such as educational attainment, labour market experience, firm and demographic characteristics or sector and occupational sorting is small and statistically insignificant, though, positive at the bottom and top. A positive value implies that gender differences in observable labour market characteristics are higher in 2020 than in 1984 in West Germany and thus that the distance of females to their male colleagues increased over time. Thus, low- and top-income earning women did not catch-up over time in terms of general observable characteristics. Gender differences in characteristics were markedly and statistically significantly reduced over time at the mean and median. Thus, our results suggest only small changes in the explained part that are not statistically different from zero at the bottom and top but negative and significant effects at the mean and median.

The representation in Table 3 splits the unexplained component in differences in prices of observable characteristics and differences in intercepts. These two parts go always in opposite directions and hence off-set or weaken the price effect. For instance, while we concluded from Table 2 that differences in prices between men and women increased over time at the top, the double decomposition suggests that differences in prices by gender

were successfully reduced at the top. The intercept part increased instead. Differences in remuneration were reduced at the mean, median and top and contributed substantially to a reduction of the gap. At the bottom, the remuneration of men compared to women for the same set of characteristics rose substantially.

As stated, interpretation of the intercept part is ambiguous in the literature (Blinder 1973; Jones 1983). Technically, this part represents gender differences in intercepts and may represent differences in starting points between men and women. This part is a major driver of the divergence of the GPG over time at the mean, median and 90th percentile and counteracts the related negative unexplained parts.

Our aggregate decomposition analysis shows that the main driver of the gap is the unexplained plus intercept component being negative at the bottom but positive at upper parts of the distribution. This finding is in line with e.g. Antonczyk et al. (2010)⁹ whose results suggest that the price effect is a main driver of the gap at the top, while it is slightly negative or not statistically different from zero at lower parts of the distribution.

To sum-up, we find that the (aggregate) GPG changed substantially between the two points in time we consider. We find a strictly increasing pattern along the distribution: the gap converged at the bottom and middle of the distribution but diverged at the top. Both the con- and divergence are statistically significant (at a

Observe that Antonczyk et al. (2010) look at the GPG between 2001 and 2006 in West Germany using linked-employer employee data.

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10% significance level). This finding implies that GPGs at the top are particularly persistent in the West-German labour market. For example, Gallego Granados and Wrohlich (2019) find that both the observed and selection-corrected wage gap converged statistically significantly at the bottom and median, while the authors found no statistically significantly change at upper parts of the wage distribution. These results suggest, similar to ours, that sticky floors were successfully reduced over time, while glass ceiling continues to persist in West Germany.

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Further, our results suggest that equal-pay legislation was not successfully implemented at the bottom. At the bottom the price effect is positive and thus weakened the trend of a closing GPG at the 10th percentile. In contrast, gender differences in remuneration for the same set of characteristics substantially decreased over time and triggered the convergence of the GPG between 1984 and 2020 for top- and mid-income earners.

Our double decomposition results immediately show that the GPG increased over time only at the top and thus that glass ceiling is the prevalent problem in West Germany. Note, however, that even though we can draw conclusions on changes in the GPG and its component over time, the coefficient estimates are only descriptive as we cannot control for general unobservable characteristics such as individual ability or motivation. Further, the estimates may suffer from sample selection bias.

Differences in labour force participation over time may also have driven the change in the GPG over time. Differences in male-female wage structures may be biased due to endogenous selection arising from nonrandom ways in which individuals select themselves into (full-time) employment. There is an increasing literature showing that sample selection affects the GPG and that it changes over time (e.g. Mulligan and Rubinstein 2008; Arellano and Bonhomme 2017; Fernández-Val et al. 2020, for the US, UK and US, respectively). Yet, none of these approaches uses unconditional quantile regressions. Hence, we can implement sample-selection correction only at the mean (using the standard Heckman (1979) two-stage model). Typically, this method uses at least one excluded regressor (i.e. an instrument) that is correlated with the probability of full-time employment but not with the hourly wage.

To correct for sample selection, Arellano and Bonhomme (2017) follow Blundell et al. (2003) and use their measure of potential out-of-work income, interacted with marital status, as excluded regressor. ¹⁰ In the spirit of their identification strategy, we use an indicator variable for asset income interacted with marital status. Yet, the first-stage regressions show that this instrument does not

work for women in 1984 and 2020 and for men in 2020. Thus, failing to identify the employment decision in our application we do not report the results in the paper.¹¹

Results from the related literature for West Germany e.g. Biewen et al. (2018) and Gallego Granados and Wrohlich (2019) find evidence for increasing employment selection in full-time work in West Germany over time. Yet, the main conclusions of Gallego Granados and Wrohlich (2019) do not change when using the selection-corrected approach compared to the results from the observed data. Nonetheless, our results can be interpreted only as associations and conditional on the sample of full-time employees in West Germany.

5.3 Detailed decomposition

The results of the aggregate decomposition shown above may be, at a first glance, at-odds with the finding in the literature that women caught-up in terms of general human capital characteristics (e.g. Goldin 2014; Blau et al. 2017). In order to find out more about changes in specific sets of controls such as human capital and labour market characteristics or the role of industrial and occupational sorting, we conduct a detailed decomposition. The detailed decomposition allows us to associate parts of the explained and unexplained component to specific covariates or sets of covariates.

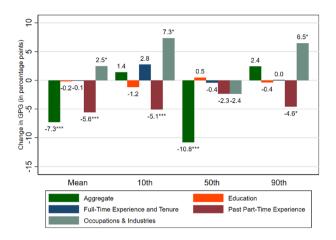
Figure 1, panel (a), shows that the change in the explained component attributable to selected sets of controls. The first bar represents the aggregate explained component that is already shown in Table 3 for convenience. The remaining bars (number of bars) correspond to changes in the explained component attributable to (2) educational attainment, (3) actual full-time experience and job tenure, (3) past part-time experience and (4) occupational or industrial sorting, respectively. Panel (b) shows the corresponding results of the unexplained component.

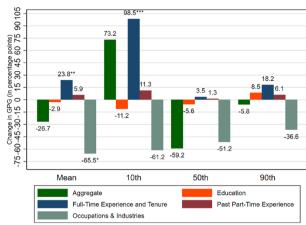
The detailed double decomposition shows that differences in education between men and women changed relatively little and thus did not markedly contribute to the convergence of the gap over time at the mean, bottom and median. Also at the top, the change is tiny, though negative and hence did not contribute to the increase in the 90th-percentile GPG over time. In contrast, gender differences in prices for education increased at the top and thus drove the divergence of the top-income GPG. At all other percentiles these differences in prices were successfully reduced and contributed markedly to the convergence of the gap.

 $^{^{10}}$ The authors estimate this variable for each individual in the sample using the Institute of Fiscal Studies (IFS) tax and welfare-benefit simulation model.

¹¹ The results are available from the authors upon request.

 $^{^{12}}$ Figure 2 in the Appendix shows the detailed double decomposition for the remaining sets of controls.





(a) Changes in Explained Component

(b) Changes in Unexplained Component

Fig. 1 Detailed double decomposition of changes in the gender pay gap 2020-1984 at the mean and selected percentiles for selected sets of controls. Estimation based on approach outlined in Sects. 3 and 4. Men in 1984 are used as reference category. Aggregate refers to the change in the aggregate explained (panel (**a**)) or unexplained (panel (**b**)) component. Education, Full-Time Experience and Tenure, Past Part-Time Experience and Occupations and Industries are the changes in the corresponding parts (explained or unexplained) attributable to these sets of regressors. Education includes dummies for highest educational attainment. Occupations and Industries include occupation (ISCO 88, 1-digit) and industry (NACE, level 1) dummies. Figure 2 shows the detailed double decomposition for the remaining sets of controls. ****p < 0.01, ***p < 0.05, **p < 0.1. Robust standard errors for the mean and bootstrapped standard errors across the distribution (500 replications) are used. SOEP sample weights used. SOURCE: SOEP v37

The small part of the change in the explained component attributable to education suggests that " the human capital part of the wage difference has been squeezed out" (Goldin 2014, p. 1094). Women are now better educated than men (cfr. Table 1) but they continue to lag in actual full-time labour market experience and job tenure. The latter is in line with e.g. Blau et al. (2017). Indeed, the part attributable to these controls did not change at the top, median and mean but increased at the bottom (2.8) percentage points or about 8% of the total gap at the 10th percentile). In terms of changes in prices to these characteristics, we find that they contributed significantly to the increase in the GPG over time at the top (18.2 percentage points given a total change in the top-GPG of 6.4% (log approximation)). At all other parts, price differences between men and women for full-time experience and job tenure increased as well, especially at the bottom.

Difference in past part-time experience are negative throughout the distribution. This finding is in line with the descriptive evidence from above and e.g. Schmitt and Auspurg (2022) suggesting that more women than men make part-time experiences and that this difference increased over time. However, gender differences in remuneration to part-time work have increased throughout and is again a main driver of the divergence of the GPG at the top.

The final bar shows the role of industrial or occupational sorting for changes in the GPG. The results show

that industrial and occupational sorting is a main driver of a positive GPG throughout the wage distribution. The latter suggests that men and women continue to select themselves in gender-specific occupations. More so, this selection behaviour has increased over time. In contrast, gender differences in remuneration to industrial or occupational sorting declined at all points. The decline is particularly pronounced at the bottom of the wage distribution and contributed thus statistically significantly to the convergence of the gap.

All in all, the results reveal substantial differences in changes in the GPG across the distribution. We find pronounced rates of convergence for West Germany for lower and median parts of the wage distribution. At the top, the GPG increased over time. This result suggests that glass ceiling is a main obstacle women are facing in the West German labour market. In fact, the GPG increased at the top (e.g. Fig. 1), while it decreased at the bottom. The latter finding is in line with papers from the related literature (Biewen et al. 2018; Gallego Granados and Wrohlich 2019). The detailed decomposition of the change in the GPG revealed that for instance women continue to be punished for larger and more frequent career breaks (positive explained and unexplained part attributable to labour market characteristics). Hence, we are far from having solved gender inequality in West Germany. The unexplained part attributable to education, full-time experience and job tenure is a main driver

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of the increase in the GPG at the top. Our results suggest that contrary to political efforts to impede career breaks of women, differences in prices to labour market experience and job tenure significantly increased over the last 36 years. The increase in gender differences attributable to these labour market characteristics are main drivers of positive GPGs in the labour market.

6 Conclusion

This paper analyzes the change in the West German GPG between 1984 and 2020. We consider both estimates at the mean and across the wage distribution and run an aggregate as well as a detailed decomposition of changes in the gap over time. Our empirical findings add to the debate of the convergence of the GPG over time that has been widely discussed in the literature (e.g. Blau and Kahn 2006; Card et al. 2013; Goldin 2014; Biewen et al. 2018). Determination of the reasons for the narrowing of the gap is of interest, especially with regard to policy implications. The results suggest a statistically significant convergence at the mean, median and bottom between 1984 and 2020 in West Germany. However, the gap increased at the top.

We find that the catching-up of women in terms of education did not contribute statistically significantly to a reduction of the GPG over time at all points of the wage distribution. The contribution is quantitatively small, though generally negative. This implies that the difference in educational attainment between men and women reduced slightly over the last 36 years. Women are still paid less for the same level of education (positive unexplained part attributable to education) at upper parts of the distribution. The latter is an important driver of the increase in the GPG at the top. Further, gender differences in remuneration for seniority and experience increased over time at all points and differences in characteristics of these controls could not be closed. This result suggests that it is pivotal to allow women to close career breaks in full-time employment and a stricter monitoring of equal pay at the workplace. For instance, the Transparency in Wage Structures Act (Entgelttransparenzgesetz) being in place since mid 2017 in Germany may be a powerful tool here. However, only few employees use their right and only few firms review wage structures (Eurofound 2021). Promoting it further should thus be on the political agenda.

All in all, our decomposition represents an intuitive and easy-to-implement approach to immediately grasp

changes in the GPG between two points in time. Additionally, it helps to identify potential drivers of the change. The approach permits policy implications to be immediately drawn based on statistical inference and by giving additional insights on the drivers of changes in wage gaps.

Appendices

A Unconditional quantile regression

The RIF-OLS regression model allows us to estimate the effect of explanatory variables X on the unconditional quantile Q_{τ} of an outcome variable Y. The RIF is estimated in quantile regressions by first calculating the sample quantile \hat{Q}_{τ} and computing the density at \hat{Q}_{τ} - that is, $f(\hat{Q}_{\tau})$ using kernel methods (Firpo et al. 2009).

This approach relies on an indicator function $\mathbb{1}\{Y \leq Q_{\tau}\}$, which takes the value of one if the condition in $\{\cdot\}$ is true, and zero otherwise. Estimates for each observation i of the RIF $\widehat{RIF}(Y; Q_{\tau})$ are then obtained by inserting \hat{Q}_{τ} and $f(\hat{Q}_{\tau})$ in the aggregate RIF function, defined as:

$$RIF(Y; Q_{\tau}) = Q_{\tau} + IF(Y; Q_{\tau})$$

$$= Q_{\tau} + \frac{\tau - \mathbb{1}\{Y \le Q_{\tau}\}}{f_{Y}(Q_{\tau})}$$

$$= \frac{1}{f_{Y}(Q_{\tau})} \mathbb{1}\{Y > Q_{\tau}\} + Q_{\tau} - \frac{1}{f_{Y}(Q_{\tau})} (1 - \tau)$$
(10)

where the RIF is the first-order approximation of the quantile Q_{τ} , and $IF(Y;Q_{\tau})$ represents the influence function for the τ th quantile. It measures the (marginal) influence of an observation at Y on the sample quantile. Adding the quantile Q_{τ} to the influence function yields the RIF. Firpo et al. (2009) modeled the conditional expectation of the RIF-regression function $E[RIF(Y;Q_{\tau})|X]$ as a function of explanatory variables X in the UQR:

$$E[RIF(Y; Q_{\tau})|X] = g_{Q_{\tau}}(X) \tag{11}$$

where a linear function $X\beta_{\tau}$ is specified for $g_{Q\tau}(X)$. The explanatory variables X contain time-varying controls like labour market experience and job tenure as well as time-constant controls like education. The average derivative of the unconditional quantile regression $E_X\left[\frac{dg_{Q\tau}(X)}{dX}\right]$ captures the marginal effect of a small

location shift in the distribution of covariates on the τ th UQ of Y, keeping everything else constant. Therefore, the coefficients β_{τ} can be unconditionally interpreted $E[RIF(Y; Q_{\tau})] = E_X [E(RIF(Y; Q_{\tau})|X)] = E(X)\beta_{\tau}.$ That is, the unconditional expectations $E[RIF(Y; Q_{\tau})]$ using the LIE allow for the interpretation of the unconditional mean. The interpretation of the conditional mean is valid only in the context of CQRs: $Q_{\tau}(Y|X) = X\beta_{\tau}^{CQR}$, where β_{τ}^{CQR} can be interpreted as the effect of X on the τ th CQ of Y given X. The LIE does not apply here; $Q_{\tau} \neq E_X[Q_{\tau}(Y|X)] = E(X)\beta_{\tau}^{CQR}$, where Q_{τ} is the UQ. Hence, β_{τ}^{CQR} cannot be interpreted as the effect of increasing the mean value of X in the UQ Q_{τ} . In UQR, the coefficients β_{τ} can be estimated by OLS in the following way:

$$Q_{\tau} = E[RIF(Y; Q_{\tau})] = E_X[RIF(Y; Q_{\tau})|X] = E(X)\beta_{\tau}$$
(12)

B Inference

The derivation of the asymptotic distribution of $\sqrt{N}\hat{\delta} = \sqrt{N}(\hat{\delta}_X\hat{\delta}_{FX}\hat{\delta}_{SX}\hat{\delta}_{FSX})$ follows the same line of argument as in Gelbach (2016). In particular, given that all estimators involved in the decomposition are asymptotically normal and given that the decomposition involves continuously differentiable functions of these estimators, joint asymptotic normality of the decomposition components follows from the delta method.

The elements of the decomposition, $\hat{\delta}$ in (7) can be written as:

$$\sqrt{N}(\hat{\delta} - \delta) = \sqrt{N}(\hat{\Gamma}\beta^{\hat{f}ull} - \Gamma\beta^{full}) \tag{13}$$

where

$$X_2 = X_1 \Gamma + W \tag{14}$$

with W, matrix $(N \times 4K)$ of the error terms. (13) can be written as:

$$\sqrt{N}(\hat{\delta} - \delta) = \hat{\Gamma}\sqrt{N}(\beta^{\hat{f}ull} - \beta^{full}) + \sqrt{N}(\hat{\Gamma} - \Gamma)\beta^{full}$$
(15)

where $\hat{\Gamma} = (X_1^{'}X_1)^{-1}X_1^{'}X_2$ and given that:

$$\hat{\Gamma} - \Gamma = (X_1' X_1)^{-1} X_1' W$$

(15) can be expressed as:

$$\sqrt{N}(\hat{\delta} - \delta) = \hat{\Gamma}\sqrt{N}(\beta^{\hat{full}} - \beta^{full}) + (\frac{X_1'X_1}{N})^{-1}\frac{X_1'W}{\sqrt{N}}\beta^{full}$$
(16)

The asymptotic variance of the vector $\hat{\delta}$ is given by:

where the consistent estimators for the matrices $Q = E[x_{1,i}x_{1,i}']$ and Γ have been already substituted in (16) by their consistent estimators: $\frac{X_1X_1}{N}$ and $\hat{\Gamma}$, respectively.

$$AsyCov(\hat{\delta}) = \underbrace{\hat{\Gamma}AsyVar(\hat{\beta}^{full})\hat{\Gamma}'}_{I} + \underbrace{\left(\frac{X_{1}'X_{1}}{N}\right)^{-1} \operatorname{plim}\left(\frac{X_{1}'W\beta^{full}\beta^{full'}W'X_{1}}{N}\right)\left(\frac{X_{1}'X_{1}}{N}\right)^{-1}}_{II} + \hat{\Gamma}AsyCov\left(\sqrt{N}(\beta^{\hat{full}} - \beta^{full}), \frac{X_{1}'W\beta^{full}}{\sqrt{N}}\right)\left(\frac{X_{1}'X_{1}}{N}\right)^{-1} + \underbrace{\left(\frac{X_{1}'X_{1}}{N}\right)^{-1}AsyCov\left(\frac{X_{1}'W\beta^{full}}{\sqrt{N}}, \sqrt{N}(\beta^{\hat{full}} - \beta^{full})\right)\hat{\Gamma}'}_{IV}$$

$$(17)$$

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Term I in (16) entails the asymptotic variance of $\beta^{\hat{full}}$ that can be consistently estimated under standard assumptions. In particular, consider the vector of all the parameters estimated from the full model $\hat{\beta} = (\hat{\alpha}^{full'})\hat{\beta}^{full'}$):

$$var(\hat{\beta}) = (X'X)^{-1}(X'\Sigma X)(X'X)^{-1}$$
(18)

where Σ is the variance covariance matrix of the error terms ϵ^{full} in the full specification (4) and $X = [X_1X_2]$. The asymptotic variance of $\hat{\beta}^{full}$ is the sub-block of $var(\hat{\beta})$ corresponding to the variables in X_2 .

By organizing the observations by groups (for instance gender and period) ϵ^{full} can be thought as $\epsilon^{full'} = (\epsilon_{FS}^{full'} \epsilon_{MS}^{full'} \epsilon_{FE}^{full'} \epsilon_{ME}^{full'})$ where F = female and M = male and S = starting period and E = ending period. It follows that

$$\Sigma = \begin{bmatrix} \sigma_{FS}^2 \mathbf{1}_{N_{FS},N_{FS}} & \mathbf{0}_{N_{FS},N_{MS}} & \mathbf{0}_{N_{FS},N_{FE}} & \mathbf{0}_{N_{FS},N_{ME}} \\ \mathbf{0}_{N_{MS},N_{FS}} & \sigma_{MS}^2 \mathbf{1}_{N_{MS},N_{MS}} & ... & ... \\ \mathbf{0}_{N_{FE},N_{FS}} & ... & ... & ... & ... \\ \mathbf{0}_{N_{ME},N_{FS}} & ... & ... & ... & ... \end{bmatrix}$$

where $\mathbf{1}_{K,L}$ and $\mathbf{0}_{K,L}$ are $(K \times L)$ matrices whose entries are all equal to one and zero, respectively, and N_{FS} is the number of observations for category F at time S. The $AsyVar(\hat{\beta}^{full})$ can be estimated consistently by taking

the appropriate sub-block of a consistent estimate of (18) where the single components in Σ are obtained from the consistent estimates of the OLS residual from the full model: $\hat{\epsilon}^{full} = Y - X \hat{\beta}^{full}$, i.e. $\hat{\sigma}_{FS}^2 = \frac{\hat{\epsilon}_{FS}\hat{\epsilon}_{FS}}{N_{FS} - K}$. The estimation of the middle part of term II in (17) can

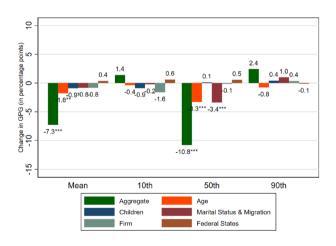
The estimation of the middle part of term II in (17) can be obtained by using the consistent estimates of β^{full} and W, i.e. $\hat{\beta}^{full}$ and $\hat{W} = X_2 - X_1 \hat{\Gamma}$. The estimation of terms III and IV requires the estimation of the covariance between $\sqrt{N}(\hat{\beta}^{full} - \beta^{full})$ and $\frac{X_1W\beta^{full}}{\sqrt{N}}$. Given standard assumption on the error terms, the consistent estimation of the covariance is given by the columns corresponding to the variables X_2 the matrix below:

$$plim \left(rac{X^{'}X}{N}
ight)^{-1} plim \left(rac{X^{1} \epsilon^{ ext{full}} eta^{ ext{full}'} W^{'} X_{1}^{'}}{N}
ight)$$

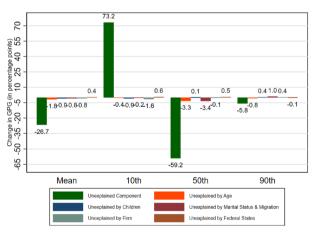
where β^{full} , ϵ^{full} and W are substituted by their corresponding consistent estimators.

C Further empirical results

See Fig. 2, Table 4 and Fig. 3.







(b) Change in Unexplained Component

Fig. 2 Detailed double decomposition of changes in the gender pay gap 2020-1984 at the mean and selected percentiles and remaining sets of controls. Estimation based on approach outlined in Sects. 3 and 4. Men in 1984 are used as reference category. Aggregate refers to the change in the aggregate explained (panel (**a**)) or unexplained (panel (**b**)) component. Age, Children, Marital Status & Migration and Firm, Federal States are the changes in the corresponding parts (explained or unexplained) attributable to these sets of regressors. Age includes age dummies, Children includes dummies for having at least one child, for having at least one child below three years and between three and six years, respectively, Firm includes small (< 200 employees) and medium (200 - 1999 employees) firms.** **p < 0.01, **p < 0.05, **p < 0.1. Robust standard errors for the mean and bootstrapped standard errors across the distribution (500 replications) are used. SOEP sample weights used. Source: SOEP v37

Table 4 Base and full OLS regressions of log hourly wages

Regression	(1)	(6)	
	Base	Full Mean	
	Mean		
Female X Year	- 0.161*** (0.029)	0.179 (0.299)	
Female	- 0.166*** (0.021)	0.010 (0.178)	
Year	- 1.513*** (0.016)	- 1.574*** (0.170)	
Full-time experience (in years)		0.022*** (0.005)	
Full-time experience squared		- 0.000*** (0.000)	
Tenure (in years)		0.007*** (0.001)	
Past part-time experience (in years)		0.005 (0.004)	
Basic secondary education (Hauptschule)		- 0.072*** (0.025)	
Upper secondary education (Abitur)		0.095*** (0.026)	
Other degree		- 0.064 (0.040)	
No degree		- 0.007 (0.041)	
Married		0.054*** (0.019)	
Migration background		0.040 (0.025)	
Civil servant		0.013 (0.039)	
Small firm <200 employees		- 0.197*** (0.029)	
Medium firm 200–1999 employees		- 0.130*** (0.021)	
At least one child		- 0.011 (0.026)	
Child <2 years		0.016 (0.033)	
Child 3–6 years		0.011 (0.035)	
Young 16–29 years		0.040 (0.065)	
Adult 30–39 years		0.045 (0.043)	
Adult 40–49 years		- 0.001 (0.030)	
Constant	3.220*** (0.013)	3.105*** (0.130)	
Observations	8202	8202	
Adjusted R-squared	0.760	0.847	
R-squared	0.761	0.852	

 $[\]Delta = \hat{\alpha}_1^{\textit{base}}$ is the coefficient estimate of Female X Year from the base and $\hat{\alpha}_1^{\textit{full}}$ is the corresponding coefficient estimate from the full regression. Female X Year is an interaction between the female and 2020-year dummy. Dummy variables if not specified differently. Secondary education (Realschule), Old (50–67) years and large firm (more than 1,999 employees) used as reference categories, respectively. Log hourly gross wage in 2015-prices. Federal state as well as occupation (ISCO 88, 1 digit) and industry (NACE, level 1) dummies included. Interaction of controls with female and year dummy, respectively, included in the full regression. Robust standard errors in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1. SOEP sample weights used. Source: SOEP v37

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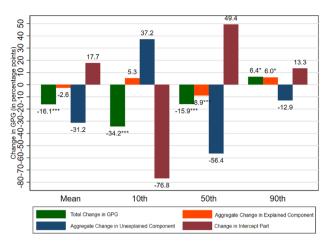


Fig. 3 Aggregate double decomposition of the change in the gender pay gap (GPG) 2020-1984 across the distribution, female reference category. Figure shows detailed double decomposition of changes in the gender pay gap (GPG) in West Germany 2020-1984. Estimation based on approach outlined in Sects. 3 and 4. Women in 1984 used as reference category. *** *p < 0.01, *** p < 0.05, *p < 0.1. Robust standard errors for the mean and bootstrapped standard errors across the distribution (500 replications) are used. SOEP sample weights used. Source: SOEP v37

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Author contributions

MBT run the analysis and was responsible for data preparation. CC derived the inference results. MBT, CC and LR interpreted together the estimates. MBT and CC were major contributors in writing the manuscript. All authors read and approved the final manuscript.

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Availability of data and materials

Access to SOEP data is subject to prior registration of researchers and their institution but is free of charge for academic purposes (see https://www.diw.de/en/diw_02.c.238223.en/soep_conditions.html for details). Yet, as data access is restricted to registered users, we cannot deposit the files in an openly accessible data repository.

Declarations

Competing interests

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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