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Invited Viewpoint

Programmable multi-layered auxetic mechanisms

Niraj Kamal Karunanidhi[1], Martin Sobczyk[2,*], Sebastian Wiesenhütter[1], Thomas Wallmersperger[2], and Jörg Rainer Noennig[1]

[1] *Wissensarchitektur - Laboratory of Knowledge Architecture, TU Dresden, 01062 Dresden, Germany*
[2] *Institut für Festkörpermechanik, TU Dresden, George-Bähr-Str. 3c, 01069 Dresden, Germany*

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ABSTRACT

The present work investigates programmable auxetic surfaces and how they can be enabled to achieve a general surface shape upon external control. To actively generate target geometries from an initial geometry, a process of non-uniform expansion or contraction as well as an alteration of local curvatures are necessary. This implies the alignment of a multiplicity of control factors. The present work suggests that auxetic mechanisms hold a high potential to achieve and simplify such alignments. As a key principle for achieving defined target forms and the required shape transitions, the study identifies the modification of the local scaling factor and the Gaussian curvature of plane surfaces. Within this work, such active surfaces are created utilizing multi-layered auxetic tessellations. To control the scaling factor and the curvature of the resulting structure, we propose different multi-layered auxetic structures comprising rotational actuators. These concepts are demonstrated for the example of kagome tessellations but can easily be transferred to other auxetic tessellations.

Introduction

Programmable surfaces are flat structures which can be actuated to actively change their shape. They offer a wide range of novel applications in architecture, such as walls that can actively change their geometrical shape and size as well as their shading and ventilation capabilities.

The control over the shape of planar structures is predestined for a wide range of new applications in the field of architectural design and building construction. In the context of this paper, this control is established by the activation of surface elements, which means the introduction of possible movements to an otherwise static structure by, for example, incorporation of actuators. By this, a programming of the surface becomes possible, which means precisely controlling those movements and changes in geometry by actuating the correct actuators needed to obtain a desired target geometry. This control of movement can be achieved by multiple ways, for

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Niraj Kamal Karunanidhi and Martin Sobczyk have contributed equally to this work.

Address correspondence to E-mail: martin.sobczyk@tu-dresden.de

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example by soft smart materials like hydrogels, shape–memory polymers or actuator-controlled mechanisms. For example, Klein et al. [1] created a systematic study of curvature change in hydrogel plates. They demonstrated, how a shape change of flat hydrogel structures can be directed by a change of humidity and plate thickness. Mao et al. [2] demonstrated the usage of combined smart hydrogels and shape–memory polymers (SMPs) for 3D-printed reversible shape changing components. Mailen et al. [3] investigated the usage of localized heating and shrinking of flat SMP surfaces into 3D shapes. A review on the topic of smart soft materials for shape changing surfaces that identified the two main strategies of bending and buckling responsible for shape shifting in flat structures and characterized the underlying mechanisms of material programming can be found in van Manen et al. [4]. Ehrenhofer and Wallmersperger [5] investigated the properties of actuated structures combining soft smart materials and more rigid structural components.

To obtain a programmable, active surface, we investigate in this study (i) stacked auxetic tessellation patterns, (ii) their arrangement and (iii) the interplay between or the number of stacked layers and the resulting controllability of the structure.

Active responsiveness of spatial structures such as walls, ceilings and partitioning enables the design of entirely new working and living environments with built-in intelligence that can lead to higher user comfort as well as to resource and energy efficiency. Interior components such as adaptive acoustic panels or flexible partitioning walls, as well as elements which control light, energy consumption or visibility through the exterior building envelope could provide a novel type of construction elements for architectural and spatial designs. Their development, however, necessitates the integration of current advancements in the fields of smart materials, smart systems research, mechanical engineering, mathematics and computer science. A review on smart materials in adaptive architecture applications is given by Sobczyk et al. [6].

Auxetic mechanisms are applied as metamaterials in various fields such as medicine, robotics, architecture and product design, and are proven to be useful for programmable surfaces. For example, Naboni and Mirante [7] used synclastic shells to develop bending-active architectural applications. A review which discusses the characterization, the design, modelling and the application of auxetic materials and structures is given by Saxena et al. [8]. The ability of auxetic mechanisms to expand and contract with a negative Poisson's ratio enables many unusual properties compared to classical materials, as discussed in Evans et al. [9]. For example, the auxetic mechanisms exhibit variable permeabilities and possess the ability to form synclastic or anticlastic curvatures. Also, they show large energy absorption and dissipation abilities. Extensive investigations on auxetics foams and their Poisson's ratio and their dissipation properties were conducted by Nazari et al. [10] and Jiang and Hu [11]. Similarly, the dissipation properties of auxetic infill latices were investigated by Simpson et al. [12]. Auxetic mechanisms, especially the ones comprising scissor linkages, have been extensively used in deployable structures, as they are able to expand or contract into a desired final shape and are also reconfigurable. However, the potential of auxetic mechanisms as an economical (mass-manufactured) solution also in architectural applications is not fully tapped yet.

Related work

Programmable surfaces using origami tessellations have been extensively studied in the past by Ron-Resch and others [13] due to their interesting geometries and structural flexibility. Here, one driving concept was the idea to equip hitherto fixed geometrical shapes with new dynamic capacities, thus turning them into active "robotic" structures.

Previous research has explored the capacity of programmable structures to form specific shapes by programming their scale factor [14–17].

Studies on auxetic mechanisms with rotating elements have shown that their contraction depends on the folding angles of the individual hinges, which can be modified [18] in order to induce shearing, bending, twisting, etc.

However, these findings only applied to the so-called "Diamond Plate" mechanism [19, 20], "Rotating Squares" [21] or "Rotating Rectangles" [22–25], which do not allow for different scaling factors at different points. It thus does not allow a higher diversity of geometries with many variations in scaling factor, and also disables re-programming [26, 27]. This means, that these structures can only attain one predetermined goal geometry based on their permanent structural configuration [18, 28]. Other research achieved a non-uniform scaling factor either by non-uniform shrinking [29, 30] or by non-uniform expansion [14, 31, 32] across the surface. This non-uniform in-plane shrinking or expansion led to internal compression stresses upon which buckling occurs after exceeding a critical threshold. This buckling is used to attain the final geometry. In these approaches, however, it is not discussed how the direction in which the surface buckles is controlled.

The present paper focuses on using multi-layered auxetic tessellations in such a way that a structure with multiple degrees of freedom can be achieved. Here, the goal is a manipulation of the local scaling factor and of the local Gaussian curvature along both the surface and its cross section. With this, a control of the bending, the buckling, and the direction of buckling of the entire structure is possible.

Auxetic surfaces

Auxetic materials are materials with a negative Poisson's ratio. This means, that a material would show an increase in thickness upon longitudinal elongation, compare Fig. 1 (top). Examples for experimental and numerical investigations on different auxetic materials were conducted by [33, 34]. Analogously, we use the term auxetic surfaces for flat structures which exhibit auxetic behaviour upon mechanical deformation. This means, that the structure will respond to a deformation in longitudinal direction with an expansion in the transverse directions, compare Fig. 1 (bottom).

In the present paper, we focus on origami tessellations to create auxetic surfaces. In the following paragraphs, the structural descriptions and analyses are given for the example of the kagome tessellation as a prototype origami tessellation since it is already well studied [35]. All the conducted steps and analyses can be transferred to most of the common auxetic tessellations. In Fig. 2, the kagome tessellation is shown in its transition from the closed to the open configuration.

A key goal of this research is to control the shape of a surface made of auxetic tessellations using actuators. Therefore, some form of actuation device needs to be implemented—in principle, this could be of various designs. Here, we want to limit our investigation to actuators controlling the angular displacement between two coaxially connected elements. We also do not limit the way how this control is accomplished, so it could be realized by implementing electric motors, hydraulic actuators, smart materials, or others. Here we suggest the application of actuator structures like the one depicted in Fig. 3.

The proposed structure consists of two stacked layers—a bottom and a top layer—each comprising four elements for the investigated kagome tessellation. Each element of one layer is connected to another element of the other layer by mechanical interlayer connections, still allowing the rotation of the elements. Additionally, the tessellation in the top layer needs to be a mirrored version of the bottom layer. Only then, a change of the relative angle

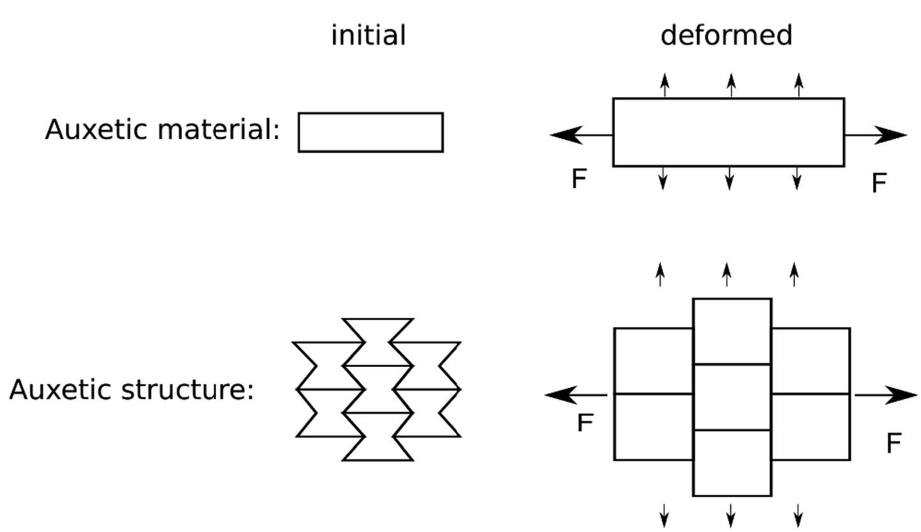

Figure 1 Auxetic materials and structures exhibit a negative Poisson ratio, and therefore show an expansion in transverse direction upon elongation in longitudinal direction.

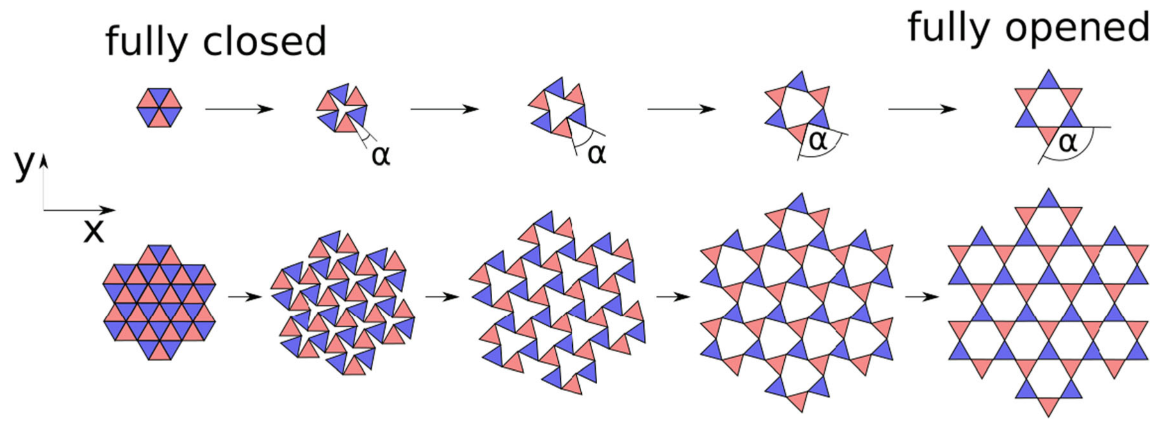

Figure 2 Kagome tessellation and its transition from the fully closed (left) to its fully open configuration (right). The colour code indicates that the blue elements will rotate in anticlockwise direction and the red elements will rotate in clockwise direction, when the structure expands.

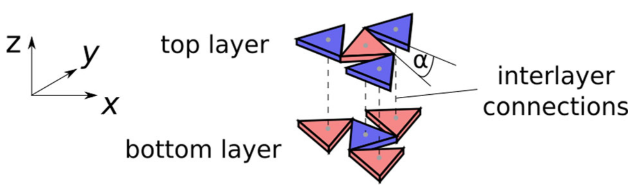

Figure 3 Example of an actuator structure. The configuration of the structure can be completely controlled by a single actuator, which changes the relative angle between a triangle element from the top and the bottom layer. The colour code indicates that the blue elements will rotate in anticlockwise direction and the red elements will rotate in clockwise direction, when the structure expands.

between an element from the top and from the bottom layer would lead to an opening or closing of the whole structure. As can be seen, the configuration of the actuator structure is completely determined for example by the angle between neighbouring triangles (angle α in Fig. 3) or by the relative angle between an element from the top and the bottom layer. This means that if one controls this angle, one controls the shape of the whole structure. Here, we denote this property of a structure as a single degree of freedom (DOF). It has to be noted, that purely translational or rotational DOFs (e.g. rigid body movements) of the whole structure are not considered by this description. Further it is noted that for the investigated structures with a single degree of freedom, the Poisson's ratio is $\nu = -1$.

One simple way to control the relative angle between an element from the top and the bottom layer would be by utilizing servo motors or other rotational actuators.

In the following, we want to investigate how an auxetic surface can be controlled by actuators to change its global shape to a desired target shape. To control the shape of a general surface, it is necessary to control the scaling factor λ and/or the Gaussian curvature K of local points on the surface.

The *scaling factor* $\boldsymbol{\lambda}$ is a quantity used to describe a linear scaling transformation of an arbitrary object in which only the size of the object is changed, but not the local angles. Therefore, the scaling factor describes an equal local expansion or contraction of the structure in all directions within the plane. As an example, the scaling factor between the fully closed configuration and the fully opened configuration of the kagome tessellation equals two, compare Fig. 2.

The *Gaussian curvature* K is another important local quantity, which describes the shape of a curved plane geometry [36, 37]. To define the Gaussian curvature, we consider an arbitrary (curved) plane in $\Re^3$. At each point of the plane, two principal curvatures are defined as the minimum and the maximum curvature of the plane at the specific point, denoted as k_1 and k_2. The Gaussian curvature (or total curvature) K is then defined as the product of the two principal curvatures [38]

$$K = k_1 \cdot k_2. \tag{1}$$

Using these two local quantities, it is possible to describe or determine the overall shape of a surface. Vice versa, K and λ can be determined for any given surface. Therefore, a shape change of an arbitrary surface can be described by a change of $K(x,y)$ and $\lambda(x,y)$ at each local point. This is true for a continuum surface but can also be approximated for a surface which is approximated by finite surface elements, as depicted in Fig. 4.

The necessary steps to create planar structures with shape-change capabilities are:

1. Definition of the initial shape and the target shape(s)
2. Discretization of the surface into finite surface elements
3. Determination of K and λ for each surface element in the initial and target surface.
4. Creation of the physical surface consisting of a planar structure, in which K and/or λ can be controlled locally by actuators
5. Mapping of the necessary inputs to the actuators to achieve a desired local K and/or λ.

The first two steps are problem specific and are not part of this investigation. The third step, the determination of K and λ for the initial and target surface can be conducted by mathematical descriptions using conformal mapping, see [36, 39, 40], and other computational methods, see [41]. In the present contribution, we focus on the creation of auxetic structures, in which we are able to control the Gaussian curvature and/or the scaling factor at local points using one or more actuators. Step five—mapping the actuator input to the desired local Gaussian curvature and/or scaling factor—is only dependent on the actuation device and the used tessellation.

In the following, we will discuss the possibilities of creating controllable planar structures, comprising auxetic tessellations. For this, we investigate two- and three-layered structures. It has to be mentioned that the structure must consist of at least two layers to achieve the control mechanisms discussed in this paper. It should be emphasized that auxetic structures are particulary suitable for this purpose because of their inherent negative Poisson's ratio.

Two-layered structures

As previously mentioned, we investigate the behaviour of multilayer auxetic structures based on the example of the kagome tessellation. To make this structure controllable using actuators, we propose to stack two kagome tessellations upon each other. Depending on how this stacking is performed, the properties of the structure—for example the number of DOFs—can be adjusted.

Here, we assume the mechanism to be based on one of the three variants as depicted in Fig. 5.

The first variant, as depicted in Fig. 5a, consists of two layers in which the triangular elements in each layer are connected at their corners (intralayer connections). Additionally, each element from one layer is connected to an element from the other layer by axles going through their respective centre points (interlayer connections). To obtain a moveable mechanism, the structure in the top layer needs to be the mirrored version of the bottom layer. This mirror transformation is explained in detail later in this section.

The second variant, as depicted in Fig. 5b, is very similar to the first variant. Here the triangular elements are also connected at their corners to their neighbouring elements in the respective layer and the top layer is again the mirrored version of the bottom layer. The difference to the first variant is in an in-plane shift of the bottom layer with respect to the top layer. This shift transformation is explained in detail later in this section. In this second variant, some of the triangular elements from the top and bottom layer do not have any interlayer connections. This means that these elements are only connected to neighbouring elements in their own layer.

The third variant, as depicted in Fig. 5c, is the so-called Hoberman mechanism [42]. In this mechanism,

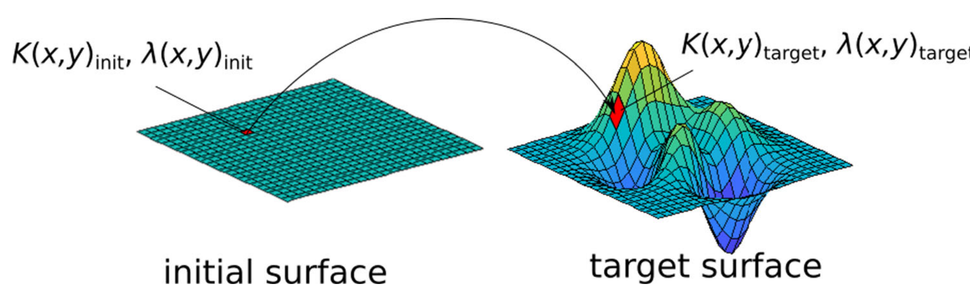

Figure 4 Shape change of an initial (flat) surface to a target geometry. For this, the Gaussian curvature K and the scaling factor λ may change at each local point or at each discrete finite surface element.

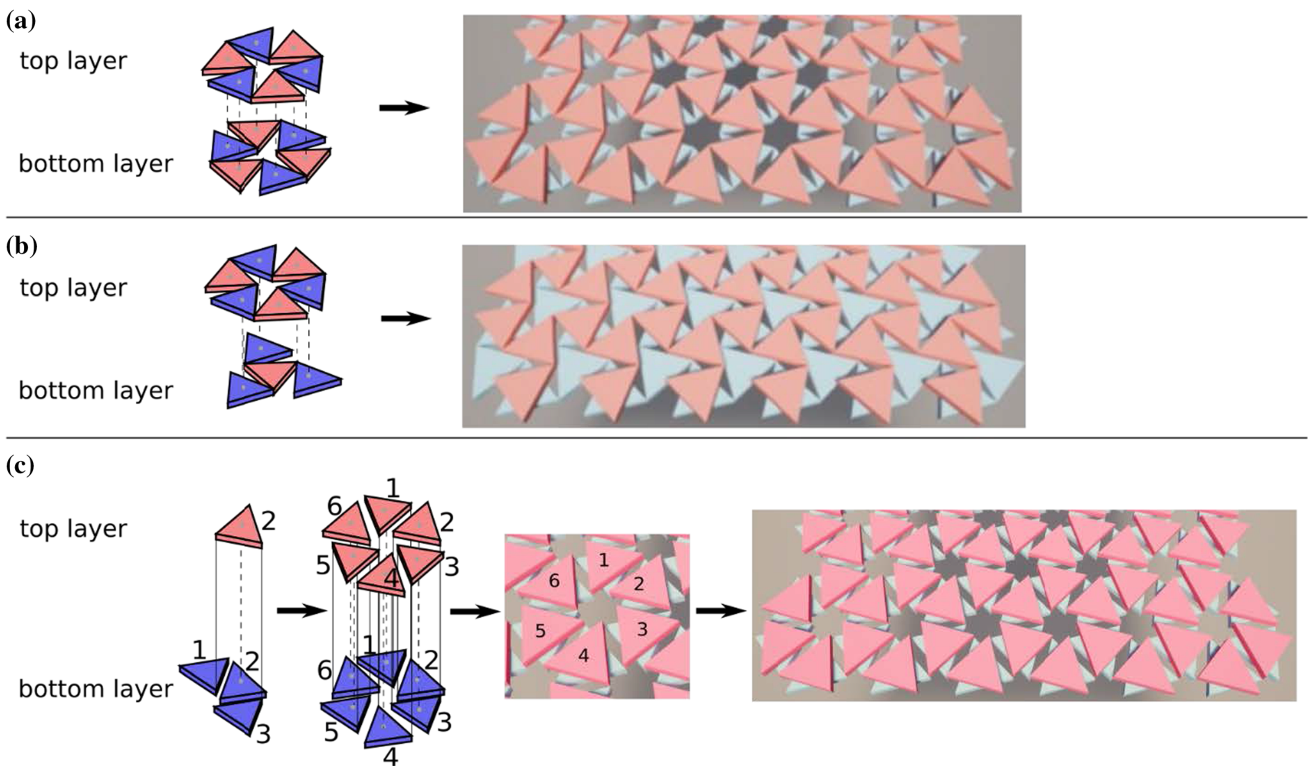

Figure 5 Basic variants of kagome-based two-layered structures. In variant **a**, the top layer and the bottom layer are mirrored, and all elements of the top have interlayer connections to the bottom layer at their centres. Variant **b** is very similar to variant **a**, but the top and bottom layer are shifted with respect to each other. Variant **c** is the Hoberman mechanism [42].

the triangular elements in each layer are not connected to their neighbouring elements in their own layer. This means that no intralayer connections exist. Each element from one layer is connected to an element from the other layer by axles going through their respective centre points. Additionally, there are interlayer connections at the corners of each element connecting to the corners from the other layer. For clarity, these connections are depicted step by step in Fig. 5c.

The variants in Fig. 5a and c only have a single DOF meaning they can be controlled with a single actuator but can attain only one preset target geometry and do not allow a wide range of different target shapes. The variant in Fig. 5b has a degree of freedom of three. The potential means of adding DOFs are discussed in the respective section further in the text. The comparison of the variants in Fig. 5a and b concerning the DOFs is shown in detail in Fig. 8.

Mirror transformation

It was already mentioned, that for the creation of variant **a** and **b** of the kagome-based two-layered structures (see Fig. 5), the top layer tessellation and the bottom layer tessellation need to be mirrored. The process of mirroring a tessellation is explained by Fig. 6. Here, three axes of rotation are depicted. If a layer A is rotated around one of these axes by 180°, the resulting state of the structure is called the mirrored layer A′ of the original state (translational shifts are neglected). An axis of rotation is found by

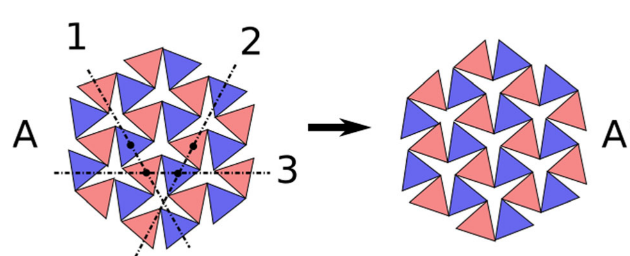

Figure 6 Schematic explanation of mirroring a tessellation. The unique axes 1, 2 and 3 are created by the connection of the centre points of neighbouring elements.

connecting the centre points of two neighbouring elements.

Shift transformation

To create variant **b** of the kagome-based two-layered structures (see Fig. 5b), the top layer and bottom layer need to be shifted with respect to each other. In the kagome tessellation, there are six meaningful shift vectors, which can be used to obtain a working mechanism. Based on the initial configuration A of the tessellation, it is then shifted in direction and magnitude of one of these possible shift vectors, see Fig. 7. Comparing the results of the shift transformation, it can be seen, that using shift vectors $\vec{a}$, $\vec{b}$, or $\vec{c}$ (= Shift1 transformation) would all result in structure B and that using the shift vectors $\vec{d}$, $\vec{e}$ or $\vec{f}$ (= Shift2 transformation) would all result in structure C. So basically, there are only these two possible results (structure B and C) after the shift transformation applied on the kagome tessellation A. Comparing the results B and C, it can be seen, that in configuration B, the red triangles are shifted to the position of the voids in the initial configuration A. In result C, the blue triangles take the position of the voids of the initial state A. Additional transformation rules can be found in kagome tessellations:

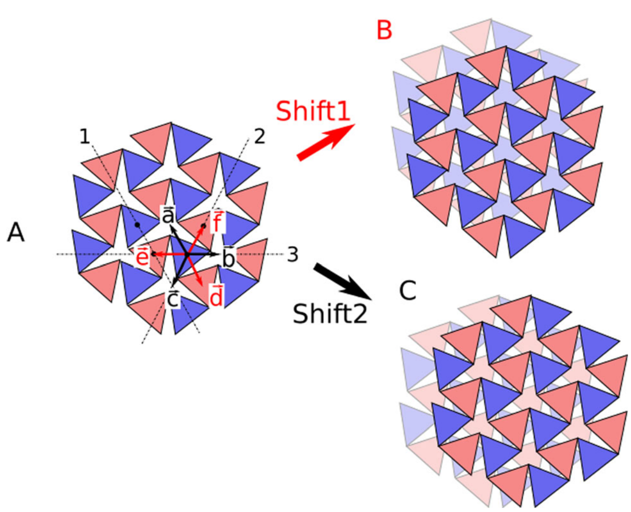

Figure 7 Schematic explanation of the shift transformation. Shifting the initial tessellation **A** in the direction of the vectors in red ($\vec{d}$, $\vec{e}$ or $\vec{f}$) would give the configuration **B**; shifting by the vectors depicted in black ($\vec{a}$, $\vec{b}$, or $\vec{c}$) would give the configuration **C**. The given axes 1, 2 and 3 are the same ones as shown in Fig. 6.

Applying the Shift1 transformation on structure B results in structure C:

$$\mathrm{Shift1(B)} = \mathrm{C} \tag{2}$$

Applying the Shift2 transformation on structure C results in structure B:

$$\mathrm{Shift2(C)} = \mathrm{B} \tag{3}$$

Shift1 and Shift2 are inverse transformations in the kagome tessellation:

$$\mathrm{Shift1(Shift2(A))} = \mathrm{Shift2(Shift1(A))} = \mathrm{A} \tag{4}$$

The mirror transformation and the shift transformation both commutate:

$$\mathrm{Shift1(A')} = (\mathit{Shift}1(\mathrm{A}))' \tag{5}$$

and

$$\mathrm{Shift2(A')} = (\mathrm{Shift2(A)})' \tag{6}$$

Introduction of additional DOFs

The mechanisms shown in Fig. 5a and c have a single DOF, with Poisson's ratio $\nu = -1$. This means in theory, that the whole structure could be actuated by one actuator. This also means, that—if the tessellation is uniformly patterned—the scaling factor upon actuation is always equal at each local point of the structure [13, 43]. Structures with a single DOF obviously cannot be used to create versatile shapes, therefore we need to introduce more DOFs to the system in order to create structures that can change their shape to more complex target shapes. In the following, we will discuss some possibilities to introduce more DOFs to a two-layered structure.

One way to introduce additional DOFs is by inserting scissor linkages. This leads to the possibility of having a different scaling factor at different points of the structure, even if the tessellations are uniformly patterned [36]. It must be noted, that by introducing more degrees of freedom, it is possible to obtain structures with locally varying Poisson's ratio. The determination of local Poisson's ratio is challenging and beyond the scope of this paper. Investigations on this effect were conducted for example by Yolcu and Baba [44].

Also, more DOFs for a structure can be obtained if the second layer experiences a certain shift transformation before being stacked upon the first layer. For example, if a layer of configuration B or C is stacked upon a layer of configuration A (see Fig. 7), a structure with multiple DOFs is formed. The reason for this is, that the kinematics of the whole structure changes depending on the configuration of the stacked layers. The kinematics can be completely described by the movement of rigid minimum sided polygons found in the structure of the system. For an example, in Fig. 8, the rigid minimum sided polygons for two different systems are depicted.

Another possibility to create controllable auxetic structures with multiple DOFs is by using a complete kagome tessellation forming one layer with many DOFs. To make this structure controllable, some of the elements from this tessellation are used to form actuator structures as shown in Fig. 3. It must be emphasized, that if all elements from the kagome tessellation layer as depicted in Fig. 3 are coaxially connected to their respective counterparts, this would end up with a structure with a single DOF. Therefore, only some of the elements may be used for that purpose. The creation of actuator structures leads to an overall structure which is two-layered since the proposed actuator structures comprise two layers. A collection of possible patterns is given in Fig. 9. Here, the triangular elements belonging to actuator structures are depicted in black, the normal triangular elements are drawn in white. Grey triangular elements represent actuator-like structures with a single DOF, see Fig. 3, but no actuator is placed in it. In other words, grey elements are connected to the bottom layer at the centre. White elements have only intralayer connections and can be inserted into the

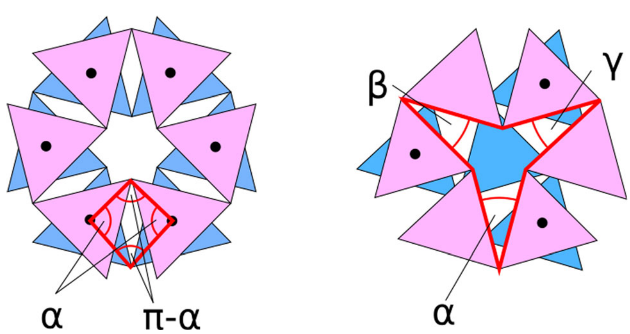

Figure 8 Illustration of the DOFs for different systems. On the left, a system comprising of the layers AA′ is depicted. Its state can be completely described by the angle α in the shown four-sided polygon and therefore has a single DOF. On the right, a system comprising of the layers AC is depicted. Its state can be completely described by the angles α, β and γ in the shown six-sided polygon and therefore its DOF equals 3.

structure to reduce the DOFs of the overall structure. It must be emphasized that the introduction of these patterns is only needed when the used actuator (and actuator like) structures have only a single DOF. Implementing these patterns is in this paper referred to a "gridding".

A main goal for gridding is to create a well-behaved structure: This means that the position and movement of every element should be determinable by the current states in which the actuators of the structure are. In order to create such structures, every element of the kagome tessellation layer should (i) belong to an actuated substructure with a single DOF or should (ii) be connected to two elements of such substructure. The more distance an element has from such substructures, the more undetermined its movement during actuation becomes. This is because the one-layered kagome tessellation has inherently multiple DOFs [45].

Variants of actuator structures

A simple actuator structure is shown in Fig. 3. If this structure is actuated—this means the relative angle between the elements of the top and bottom layer is changed by an externally controlled rotational actuator—the structure expands (opens) or contracts (closes) with a Poisson's ratio $\upsilon = -1$. This could be interpreted as a change in the scaling factor for this structure. Using these actuator structures in patterns as described in Fig. 9 would lead to an overall structure in which the scaling factor can vary locally. If there is a non-uniform scaling occurring in a plane structure, an internal compression or tensile force acts within the structure. If the structure is thin and flat, it can be approximated using plate theory. When internal compression stresses act, the plate will first remain flat and be simply compressed. Upon exceeding a critical compression force, according to plate theory, buckling occurs—therefore the previous plane geometry will snap-through to a non-plane geometry. A demonstration of this behaviour was shown by Van Manen et al. [4], a numerical investigation on smart material snap-through was conducted by Ehrenhofer and Wallmersperger [46]. For more information on the principles of buckling in plates see [47]. In the normal case, the direction of buckling is random, which limits the use cases. One possibility to direct the buckling is for example to use different elastic moduli of the layers. This would lead

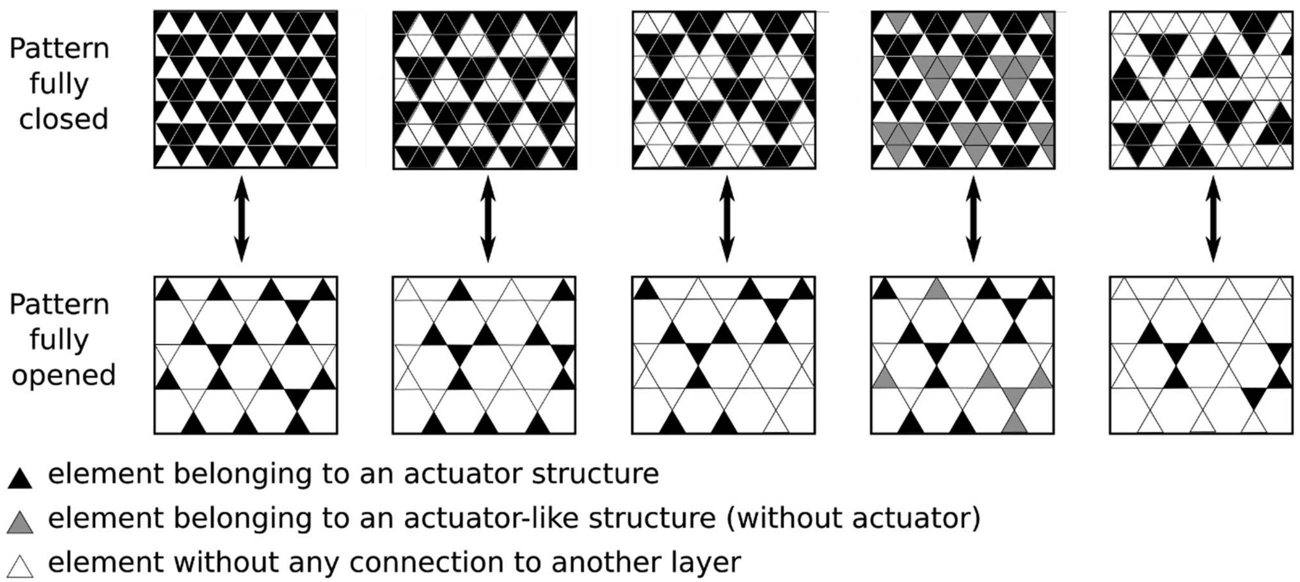

Figure 9 Schematic representation of controllable auxetic structures with multiple DOFs in closed and open configuration. Here, black triangles represent elements which are attached to an actuator structure, grey elements are attached to actuator-like structures without an actuator and white elements are triangular elements without any attachment to a second layer.

to a buckling in the direction of the layer with lower elastic modulus. Another possibility is to direct the buckling with the help of external forces like the gravity force.

A different variant for an actuator structure is depicted in Fig. 10. This structure is again composed of two layers, each containing four elements. Each element is coaxially connected to its counterpart in the other layer. Different from the first variant, the elements are not connected to their neighbouring elements within their own layer by one corner, but only by a point connection. This point connection permits the elements to only perform an in-plane rotation, but to also to perform an out-of-plane displacement. Also, the structure in the top layer and the bottom layer is both equally arranged. This means that the top layer is not the mirrored version of the bottom layer which is the case in the actuator

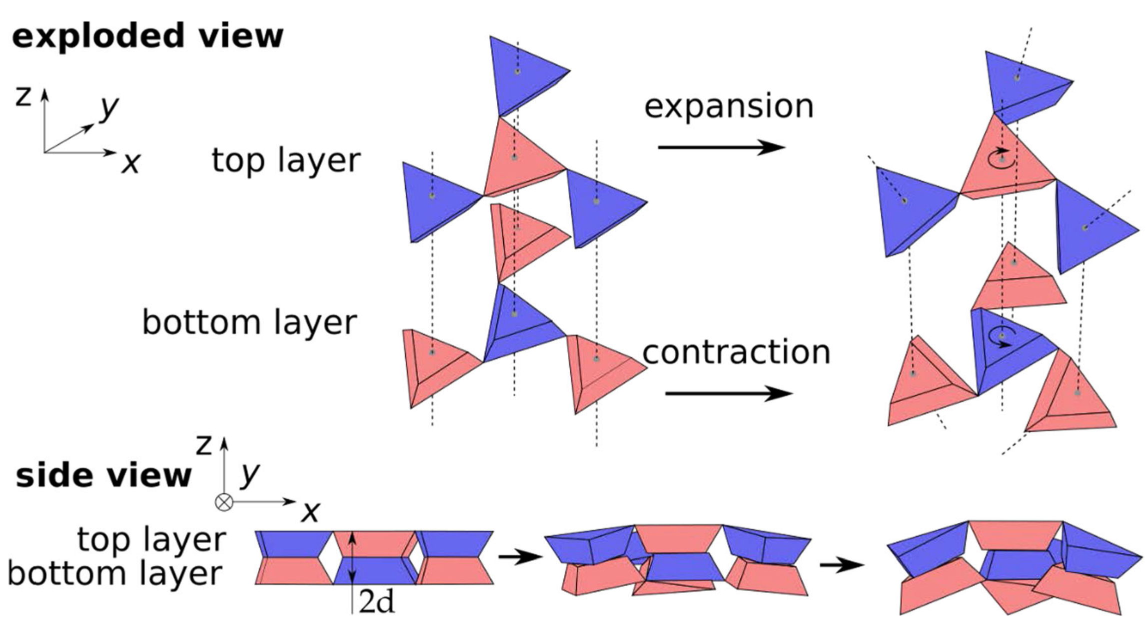

Figure 10 3D representation of a two-layered actuator structure AA which will bend upon actuation.

structure from Fig. 3. In Fig. 10, both the top and bottom layer structure (if not connected to the other layer) expands if the centre element rotates in the clockwise direction and contracts if the centre element rotates in anticlockwise direction.

Upon changing the relative angle between the top and bottom centre element using an actuator would therefore lead to an expansion of the top layer and a contraction of the bottom layer or vice versa. The overall actuator structure would therefore not expand or contract, but only bend. This behaviour has already been demonstrated by using multi-layered soft gels [4]. Using the proposed actuator structure allows changing the Gaussian curvature but not the scaling factor. Since a change of curvature is always linked to a bending moment, this actuator structure can be seen as a structural element inducing a bending moment on its surrounding.

An important difference in behaviour of the structures depicted in Fig. 10 to the structures in Figs. 3, 8 and 9 is that if top and bottom layer expand, for example by external forces, the coaxially connected elements rotate in the same direction. We denote this relation between the two layers, such as in Fig. 10, as "+". Whereas, in the previously described structures in Figs. 3, 8 and 9, coaxially connected elements rotate in opposite directions when the top and bottom layer expands. We thus denote the relation between the two layers as "−". Thus, the relations "+" and "−" between two layers describe the direction of rotation of any two coaxially connected elements, "+" being the same direction, and "−" meaning opposite directions of rotation during expansion of both layers, see Fig. 11. These relations for different layer pairings are summarized in Table 1. Here, the layers B and C denote shifted layers of layer A, compare Fig. 7. The notation A′, B′ and C′ denote the mirrored layers of A, B and C, respectively, compare Fig. 6. With layer pairing we mean the combination of two layers ("layer one" and "layer two" in Table 1) upon stacking.

From Table 1, it can be inferred that two layers with the relation "+" will behave in a similar manner as depicted in Fig. 10 and the ones with "−" as depicted for example in Fig. 3.

A third variant for an actuator structure is depicted in Fig. 12. This structure comprises three different layers, arranged in specific patterns. To describe these patterns, we use the notation AA′A (top left) and A′AA (top right) for the respective arrangements shown in Figure. The arrangement A′AA for example describes that the bottom and middle layer are of pattern A, and the top layer is of pattern A′. Pattern A denotes the basic kagome pattern as shown in Fig. 2, whereas pattern A′ denotes the mirrored version of pattern A as depicted in Fig. 6. Using these arrangements, it is possible to incorporate two rotational actuators, each controlling the relative angle between the respective elements of neighbouring layers. With these, it is possible to control the Gaussian curvature and the scaling factors of the actuator structure, compare Fig. 12 (bottom). It has to be noted that the movement of these actuator structures also depends on the thickness d_1 and d_2 in Fig. 12, which describes the vertical distance between the point connections of stacked elements. The intralayer connections within the middle layer may not hinder the rotation or transfer a bending moment. This could for example be realized by ball joints or elastic hinges.

Three-layered structures

As was discussed in the previous section, the simultaneous control of scaling factor and Gaussian curvature is only possible for the structures considered in this work if actuator structures made of three layers are utilized. In this section we want to investigate auxetic structures composed of three stacked layers made of different auxetic tessellations. In these structures, the mentioned three-layered actuator structures can be easily incorporated. In Fig. 13, the stacking of different tessellations is shown, which results in useful two- and three-layered structures. It has to be noted that the relations discussed in the section in two-layered structures listed in Table 1 are also valid here between any two stacked layers. For example, the structure AAA will not have a control over scaling factor as there are no layer combinations with a "−" relation.

When stacking different layers, the bottom layer (layer 1) would always be layer A (the standard kagome tessellation). On top of layer 1, a mirrored tessellation A′ (see Fig. 6), the same tessellation A, the shifted tessellation B (see Fig. 7) or the shifted and mirrored tessellation B′ can be stacked. By incorporating rotational actuators into these structures according to the two-layered actuator structures mentioned before, it is possible to control the scaling factor or the Gaussian curvature in these structures. If a third layer is added, it is possible to incorporate

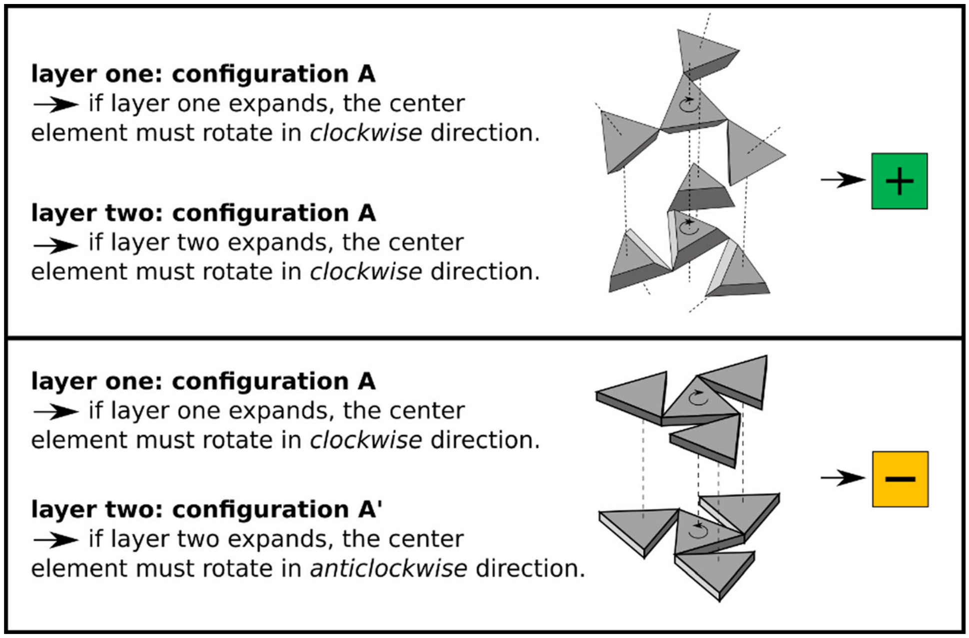

Figure 11 Graphic explanation for the different types of layer pairings with the relations "+" and "−", using the example of AA layer pairing (top) and AA′ layer pairing (bottom). If upon expansion stacked elements rotate in equal directions, the layer-pairing relation is "+", if they rotate in opposite directions, the layer-pairing relation is denoted as "−". Layer pairings with equal layer-pairing relations show similar behaviour upon actuation.

rotational actuators to form the aforementioned three-layered actuator structures, which would allow for a control of both the scaling factor and the Gaussian curvature. In Fig. 13 only a selection of useful combinations is given for the three-layered structures, which are able to change the scaling factor and the Gaussian curvature at local points.

Conclusion

The goal of this work was to introduce the idea of general controllable surfaces and to investigate possibilities on how to create these. With controllable or programmable surfaces we mean surface structures which can adapt to a variety of target shapes. For this, we explicitly investigated origami-inspired tessellations in order to achieve non-developable surfaces. Within the scope of this work, we investigated kagome tessellations as a base. This can also be extended to diamond plate or other tessellations. A multitude of different active structures could be developed using this concept.

To achieve the beforementioned goal of creating controllable surface structures, a multitude of investigations was performed: first, we identified the curvature and the scaling factor as the necessary local target variables on the surface. Also, we note that for achieving general target shapes, there need to be enough degrees of freedom to perform the necessary transformation. Possibilities to vary the number of degrees of freedom are gridding and the variation of the layer pairings. In the second step, we discovered that the mentioned target variables can be controlled by stacking two tessellation layers upon each other. The layers are connected to each other by interlayer connections. To control the degrees of freedom of the overall structure, actuators or acturator structures need to be incorporated into the overall structure. In the paper, three possibilities are given on how to design such structures. It was found that using only two stacked tessellations, only either curvature OR scaling can be controlled at one point. Whether scaling or curvature can be controlled depends on the type of the used layer pairings. Shift and mirror transformations are introduced to describe these layer pairings. If only the local scaling factor is externally controlled, the structure can undergo two different types of shape transition: if the scaling factor is changed equally at each point in the structure, this will lead to an expansion of the structure, without any bending, so the structure remains in-plane and only changes its size. If the scaling factor is changed differently at different points in the structure, this can lead to compressive stresses in the structure. If the compressive stresses surpass a threshold, buckling or snap-through of the structure occurs. The direction of the snap-through could be guided by external forces or by designing the structure in a way, that there is a preferential direction for it. A two-layered structure

Table 1 Relation between layer pairings

		Layer one					
		C'	B'	A'	C	B	A
Layer two	A	+	+	-	-	-	+
	B	+	-	+	-	+	
	C	-	+	+	+		
	A'	-	-	+			
	B'	-	+				
	C'	+					

The relations "+" and "–" between two layers denote the direction of rotation of two coaxially connected elements, "+" being the same direction, and "–" meaning opposite directions during expansion of both layers. Here, the layers B and C are shifted layers of layer A, compare Fig. 7. The notation A′, B′ and C′ describe the mirrored layers of A, B and C, respectively, compare Fig. 6. With layer pairing we mean the combination of any two layers upon stacking

can also be set up in a way, that only the Gaussian curvature can be externally controlled. By this, the out-of-plane movement can be controlled, but the overall size of the structure cannot be manipulated.

To overcome the shortcomings of two-layered structures and control scaling and curvature at the same point, three layers can be used. For this it is required, that suitable layer pairings are combined. Then, one layer pairing is used to curvature control, while the second layer pairing is used for scaling control. Three-layered structures are more challenging to manufacture. Especially the interlayer connectors pose another major difficulty in terms of mechanical/material manufacturing of the structures.

Outlook

The present work is intended to contribute to a novel approach in architectural design and engineering. It understands itself as a part of a more comprehensive research venture, which brings together different

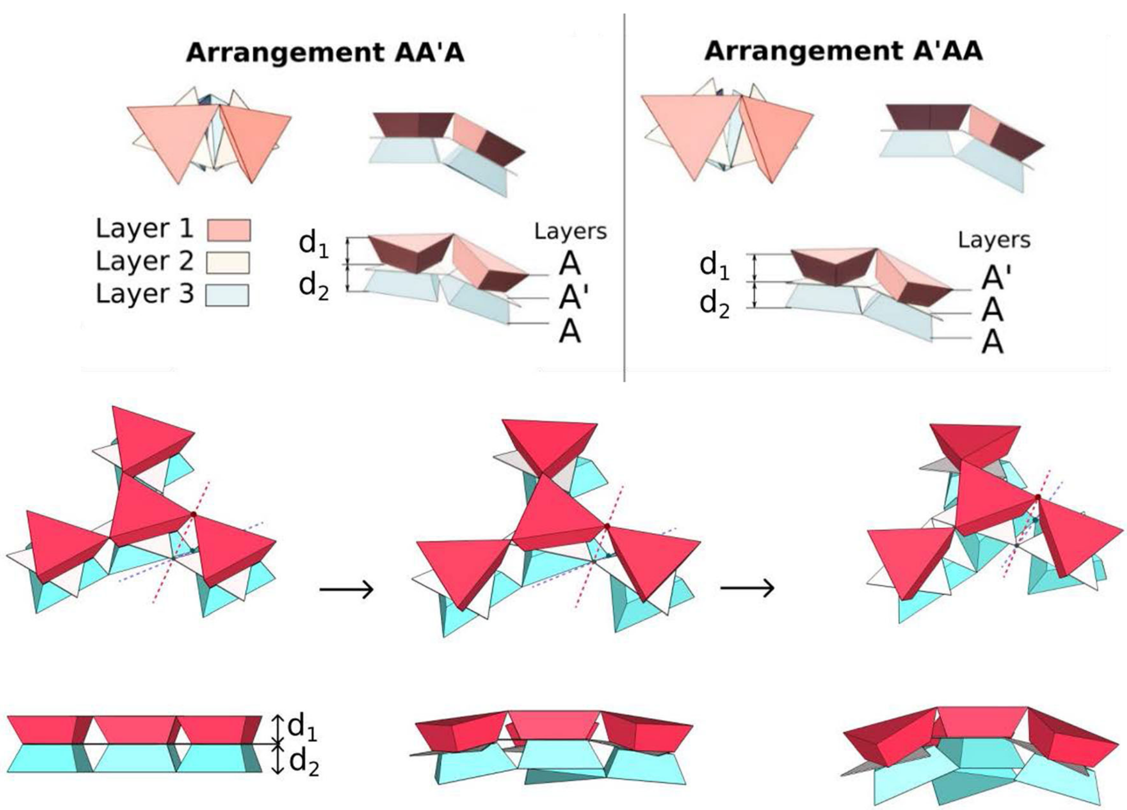

Figure 12 3D representation of a three-layered actuator structure. Here the different colours are used to distinguish between elements of the three different layers.

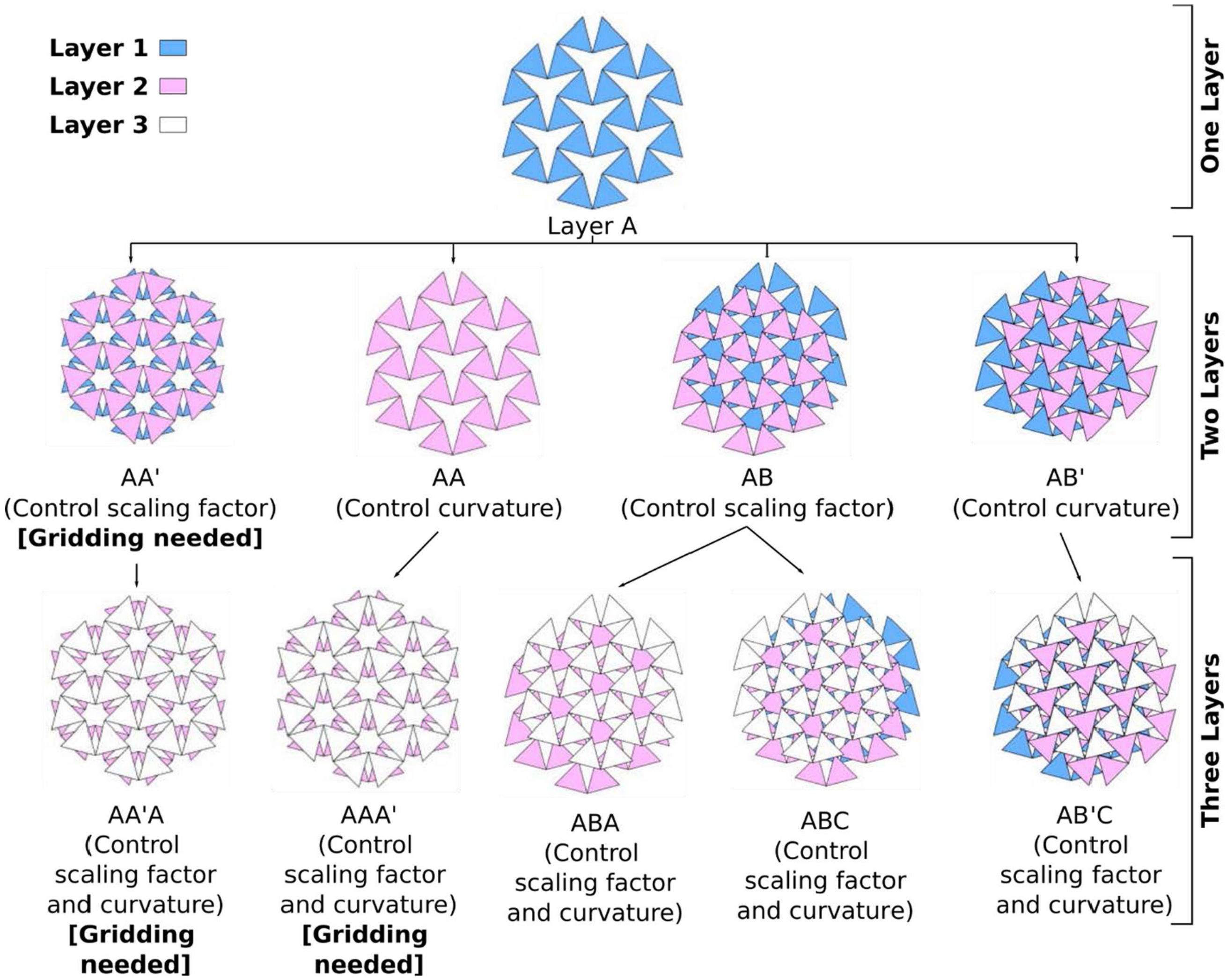

Figure 13 Some possibilities of stacking different tesselations/layers in order to obtain controllable auxetic structures for single-, two- or three-layered systems. Here we show examples of two-layered and three-layered structures in the second and the third row, respectively.

scientific expertise. It aims at spatial structures that can respond to highly volatile conditions—both in the rapidly changing human and natural environment.

Facing several unforeseen disruptions in the physical and social, global and local context, active, programmable structures could contribute to ground-breaking new solutions. These solutions range from buildings physics (adaptive light control, ventilation, acoustics) up to proactive structures that provide for more security, stability, robustness and resilience.

To achieve real-world applications, further research has to be carried out. For example, the practical limitation in terms of size and scale and materiality need to be considered. Since this paper discusses only the theoretical connections and the arrangements in multi-layered mechanisms, the practical ways of applying such procedures in corresponding mechanisms with soft/rigid elements must be studied. The mechanical details and fabrication techniques of the hinges and connections discussed in the mechanisms in the present paper must be worked out. The mechanisms discussed in this paper also demand for computational design and mapping of multi-layered auxetic mechanisms based on curvature, scaling factor and interlayer distance, which is beyond the scope of this paper. Estimating the rotation angles of individual elements to attain any geometry is substantial for programming the geometry onto such surfaces. The description of dynamic and transformative structures still poses a challenge due to limited analytical and numerical models.

Controlling the movements of flat structures has many potential applications in various fields beyond architecture, for example space- and biotechnologies. In this paper, we predominately discussed about the kagome tessellation, but a generalization for all auxetic mechanisms needs to be done mathematically. These auxetic mechanism can also be implemented into different geometries other than flat structures, for example for cuboids, fulleroids, rhombododekahedrons and others [48, 49]. We discussed some possible auxetic tessellations and some of their variants created by application of different transformations. It is necessary to further investigate the resulting structures created by the number of possible shift transformations and their respective characteristics.

The investigated auxetic structures involve a large number of rigid elements and hinges, increasing their complexity. Such hinges and mechanisms are already discussed in deployable structures [50–52]. One way to simplify these mechanisms is the usage of monolithic elastic surfaces with elastic hinges or bodies.

The options to optimize these patterns need to be further explored. Also, the ways of actuation, deployment and fabrication need to be investigated. For this, smart materials might be advantageous over classic materials.

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Author contributions

NKK contributed to investigation, methodology, visualization, conceptualization, writing—original draft, writing—review and editing. MS contributed to investigation, methodology, project administration, conceptualization, writing—original draft, writing—review and editing. SW contributed to investigation, methodology, conceptualization, writing—original draft, writing—review and editing. TW contributed to resources, project administration, conceptualization, writing—review and editing. JRN contributed to funding acquisition, resources, project administration, conceptualization, writing—review and editing.

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Data and code availability

Not Applicable.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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